

# E9: 309 Advanced Deep Learning

2-11-2020





# Housekeeping

## \* Attendance

✓ We will use the recorded sessions for attendance

★ If you are unable to attend live sessions (due to network or other issues, please indicate by email before or after class to the instructor and copy the FAs).

## \* Mid-term exam

➔ 1st week of Dec. (Modules I and II).



# Housekeeping

## \* 1st mini-project

### ✓ Deadlines

- ★ Abstract submission deadline (Nov 2nd, Monday)
  - ★ Using the google form given in the webpage
- ★ Solo projects or 2-member projects
  - ★ Indicate roles of each member in 2-member project
  - ★ 200 page abstract of the work. If modifications are needed, we will review and let you know in 2-3 days.



# Housekeeping

## ✳ 1st mini-project

### ✓ Deadlines

- ★ Report and presentation slides (Nov 19, 10 AM).
  - ★ 1-page pdf with second page only for references and tools used (Template will be provided).
- ★ Report - Indicate prior work, technical details and your contribution. Strictly adhere to the guidelines given in the template.
- ★ Slides (max 4 slides) - 4 min presentation for solo project and 6 min. for two member teams. 3 mins for your presentation and 1 min for Q&A.
- ★ Two slots are available on 2 days (pick the suitable based on your other class schedules).





# Recap of previous class





# State of affairs

- \* Encoder-decoder models with attention.

  - self attention and multi-head attention

- \* Transformer models - Introduction

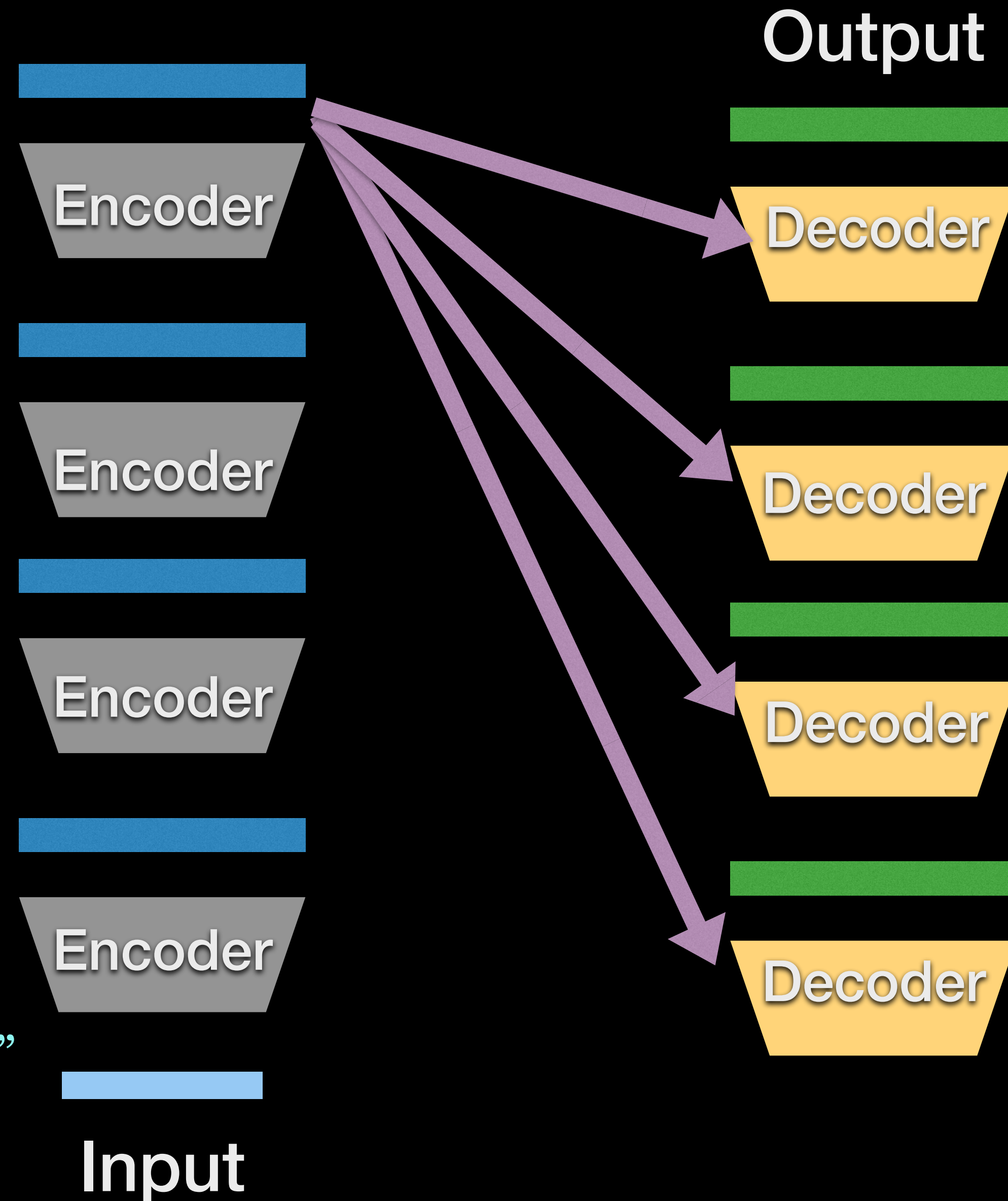




# Transformers

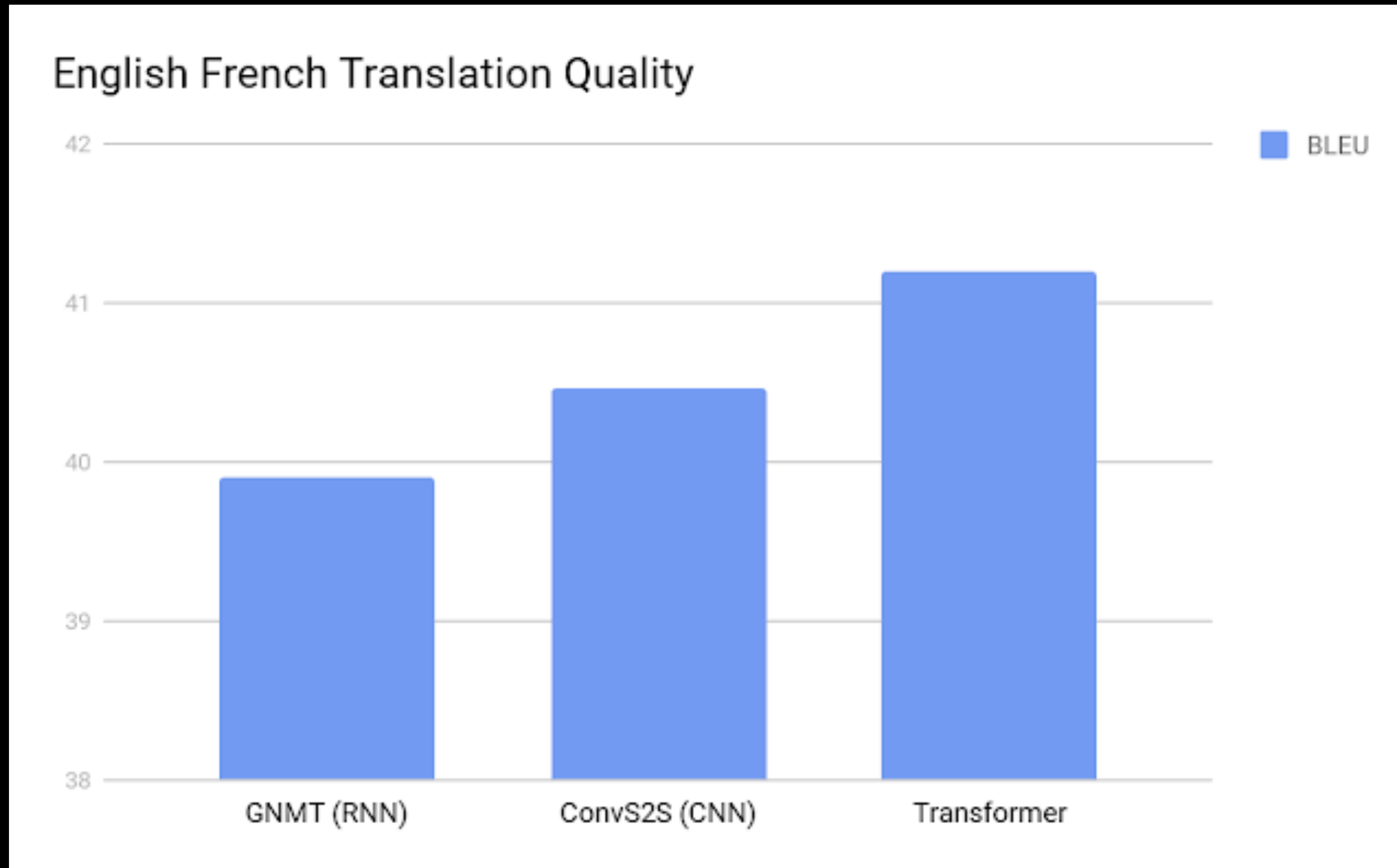
- \* Encoder Decoder architecture based models.
- \* Uses only feed forward architectures with self-attention.
- Multi-head self attention.
- \* All the encoder layers and the decoder layers have the same set of operations.

Reading Assignment - "Attention is All You Need"  
<https://arxiv.org/pdf/1706.03762.pdf>



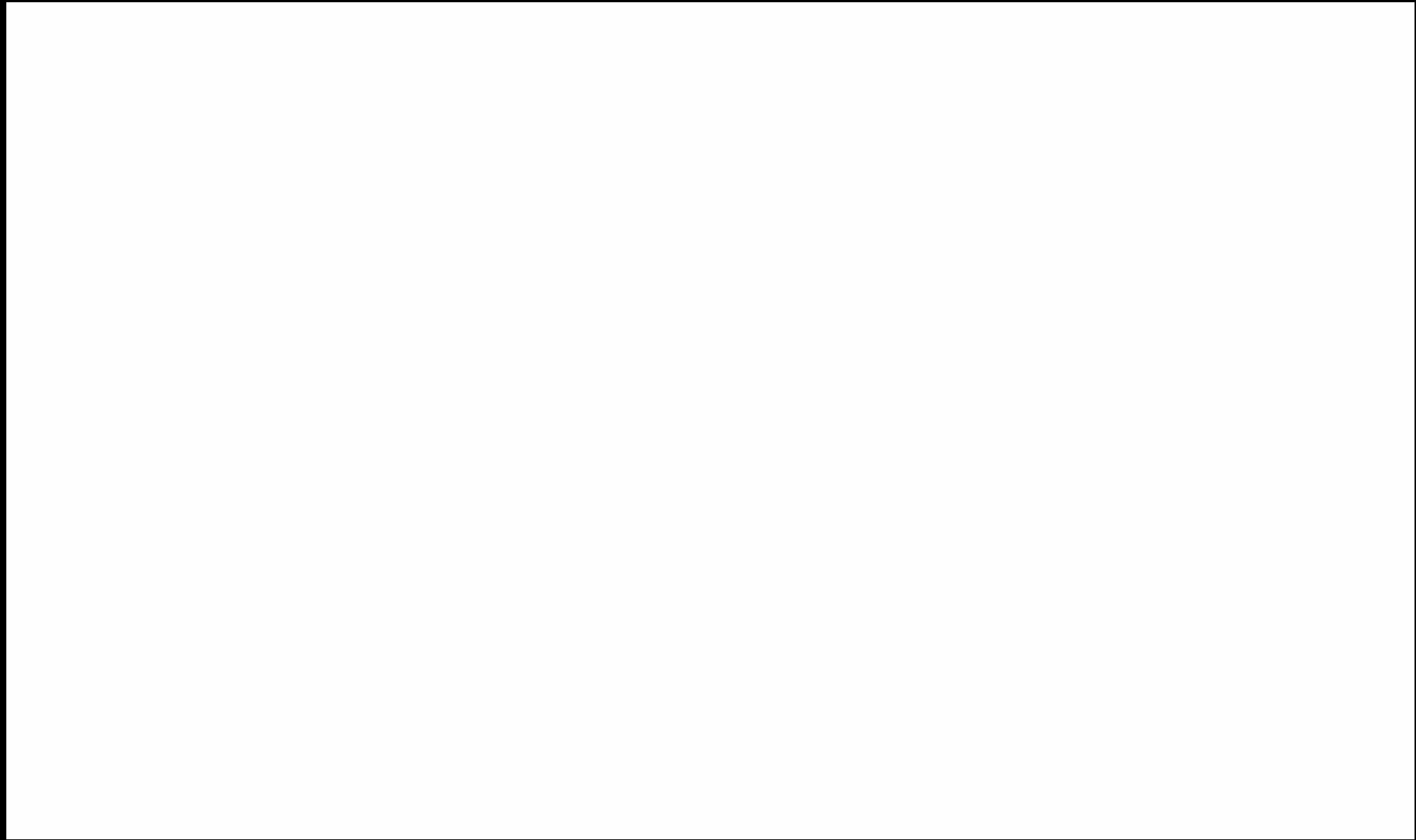


# Transformers - the state of art in NMT





# Transformers - the state of art in NMT





# Transformers

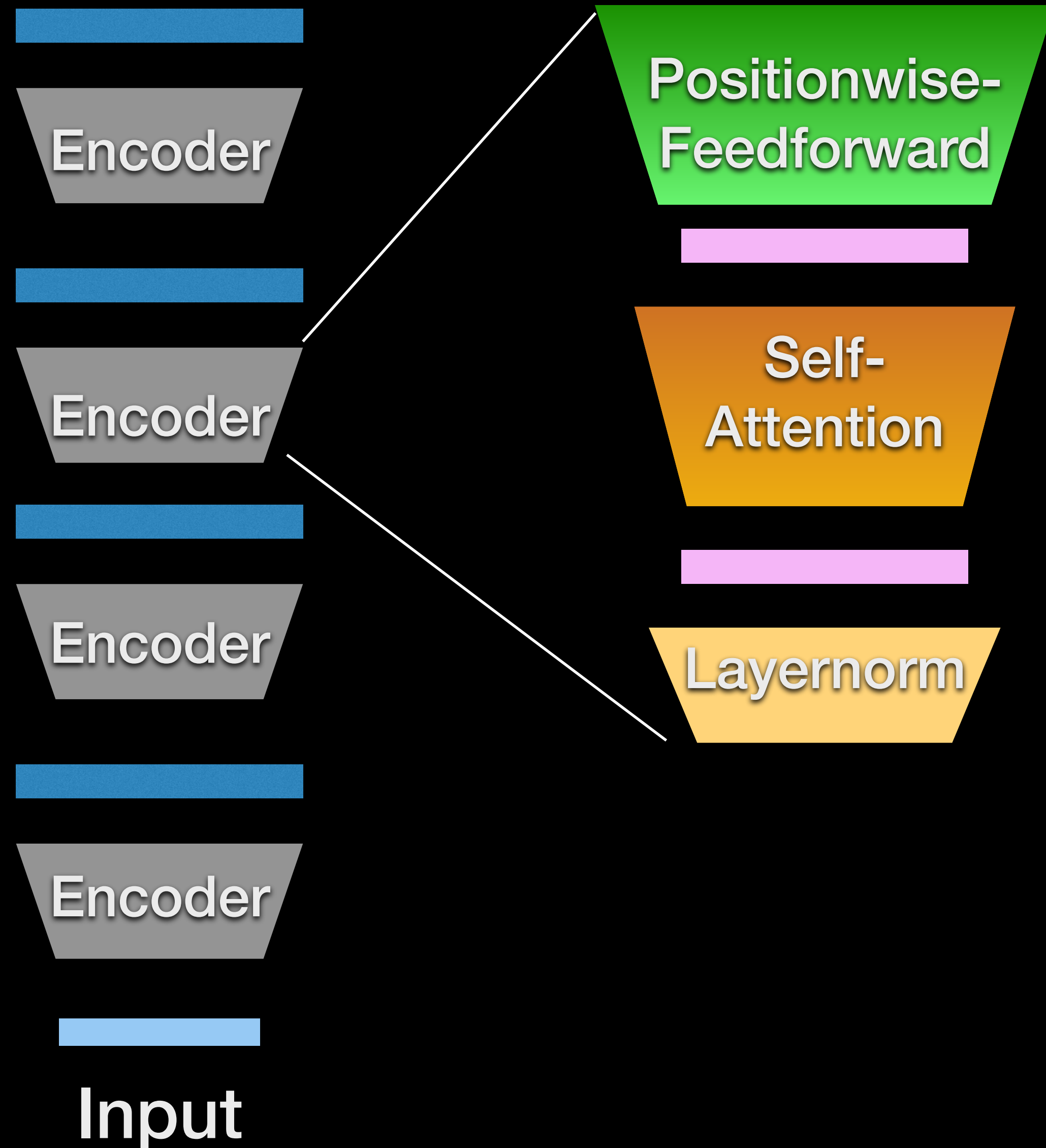
## \* Encoder layers

→ Consist of layer norm

→ Self attention (multi-head)

→ Positionwise feedforward

✓ May also consist of skip connections.





# Transformers - encoder

\* Let  $\mathbf{x}(1) \dots \mathbf{x}(T)$  denote the input and let  $\mathbf{e}^l(1) \dots \mathbf{e}^l(T)$  denote encoder outputs at layer  $l$ .

$$\bar{\mathbf{E}}^{l-1} = \text{Layernorm}([\mathbf{e}^{l-1}(1) \dots \mathbf{e}^{l-1}(T)]^T) \in \mathcal{R}^{T \times D}$$

\* Definition of layer norm

$$\text{Layernorm}(\mathbf{e}^l(t)) = \frac{\alpha^l}{\sigma_{\mathbf{e}^l(t)}} \odot (\mathbf{e}^l(t) - \boldsymbol{\mu}_{\mathbf{e}^l(t)}) + \boldsymbol{\beta}^l$$





# Transformers - encoder

Query, Key and Value

encoder layer

$T \times H$

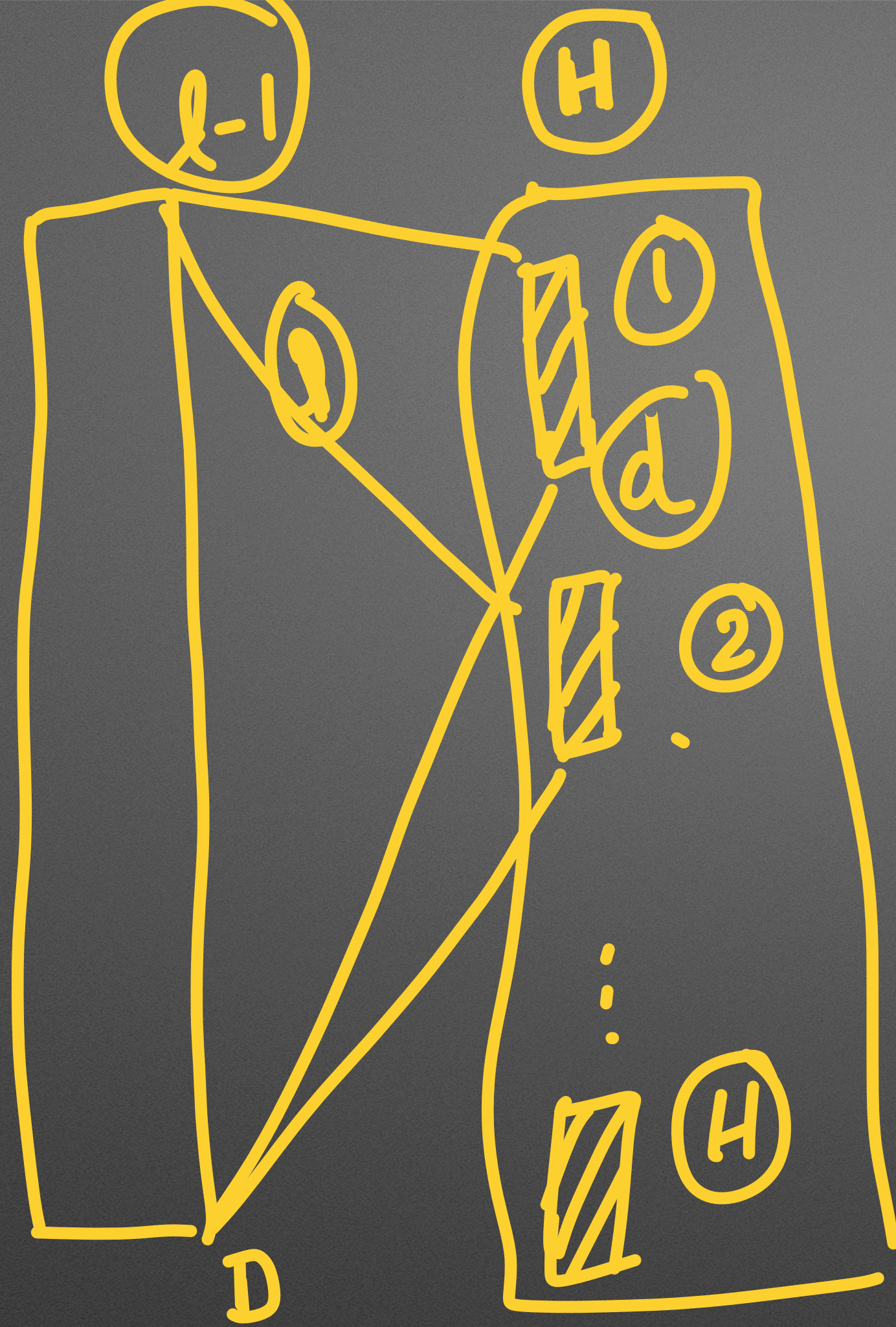
$$\begin{aligned} \vec{Q}_h^l &= \vec{E}^{l-1} \mathbf{W}_h^{l,Q} + \mathbf{1} (\mathbf{b}_h^{l,Q})^T \in \mathcal{R}^{T \times d} \\ \vec{K}_h^l &= \vec{E}^{l-1} \mathbf{W}_h^{l,K} + \mathbf{1} (\mathbf{b}_h^{l,K})^T \in \mathcal{R}^{T \times d} \\ \vec{V}_h^l &= \vec{E}^{l-1} \mathbf{W}_h^{l,V} + \mathbf{1} (\mathbf{b}_h^{l,V})^T \in \mathcal{R}^{T \times d} \end{aligned}$$

$$\mathbf{W}_h^{l,Q}, \mathbf{W}_h^{l,K}, \mathbf{W}_h^{l,V} \in \mathcal{R}^{D \times d} \quad \mathbf{b}_h^{l,Q}, \mathbf{b}_h^{l,K}, \mathbf{b}_h^{l,V} \in \mathcal{R}^{d \times 1}$$

$$h = \{1..H\} \text{ heads} \quad d = \frac{D}{H} \quad \mathbf{1} \in \mathcal{R}^{T \times 1} \text{ all ones}$$







$\ell$   
→

$$\mathcal{D} = \underline{\underline{d \times H}}$$



# Transformers - encoder

$T = 100$

## \* Multi-head attention

$$\hat{\mathbf{A}}_h^l = \mathbf{Q}_h^l (\mathbf{K}_h^l)^T \in \mathcal{R}^{T \times T}$$

*(Handwritten notes:  $T \times d$  above  $\mathbf{Q}_h^l$ ,  $T \times d$  above  $\mathbf{K}_h^l$ )*

$$\tilde{\mathbf{A}}_h^l = \text{softmax}\left(\frac{\hat{\mathbf{A}}_h^l}{\sqrt{d}}\right)$$

*(Handwritten notes:  $\mathcal{R}^{T \times T}$  in a circle next to the equation)*

$$\mathbf{C}_h^l = \tilde{\mathbf{A}}_h^l \mathbf{V}_h^l \in \mathcal{R}^{T \times d}$$

$$\left( \mathbf{A}_h^l \perp \mathbf{A}_{h+1}^l \right)$$

$\mathbf{A}_h^l \mathbf{A}_{h+1}^{lT} \approx \mathbf{I}$

## \* Context vector from self-attention

$$\mathbf{C}^l = [\mathbf{C}_1^l \dots \mathbf{C}_H^l] \in \mathcal{R}^{T \times D}$$





# Transformer - encoder



\* Position wise feedforward layer

$$\mathbf{E}_{ff}^l = \text{ReLU}(\mathbf{C}^l \mathbf{W}_{ff}^l + \mathbf{1} \mathbf{b}_{ff}^T) \in \mathcal{R}^{T \times d_{ff}}$$

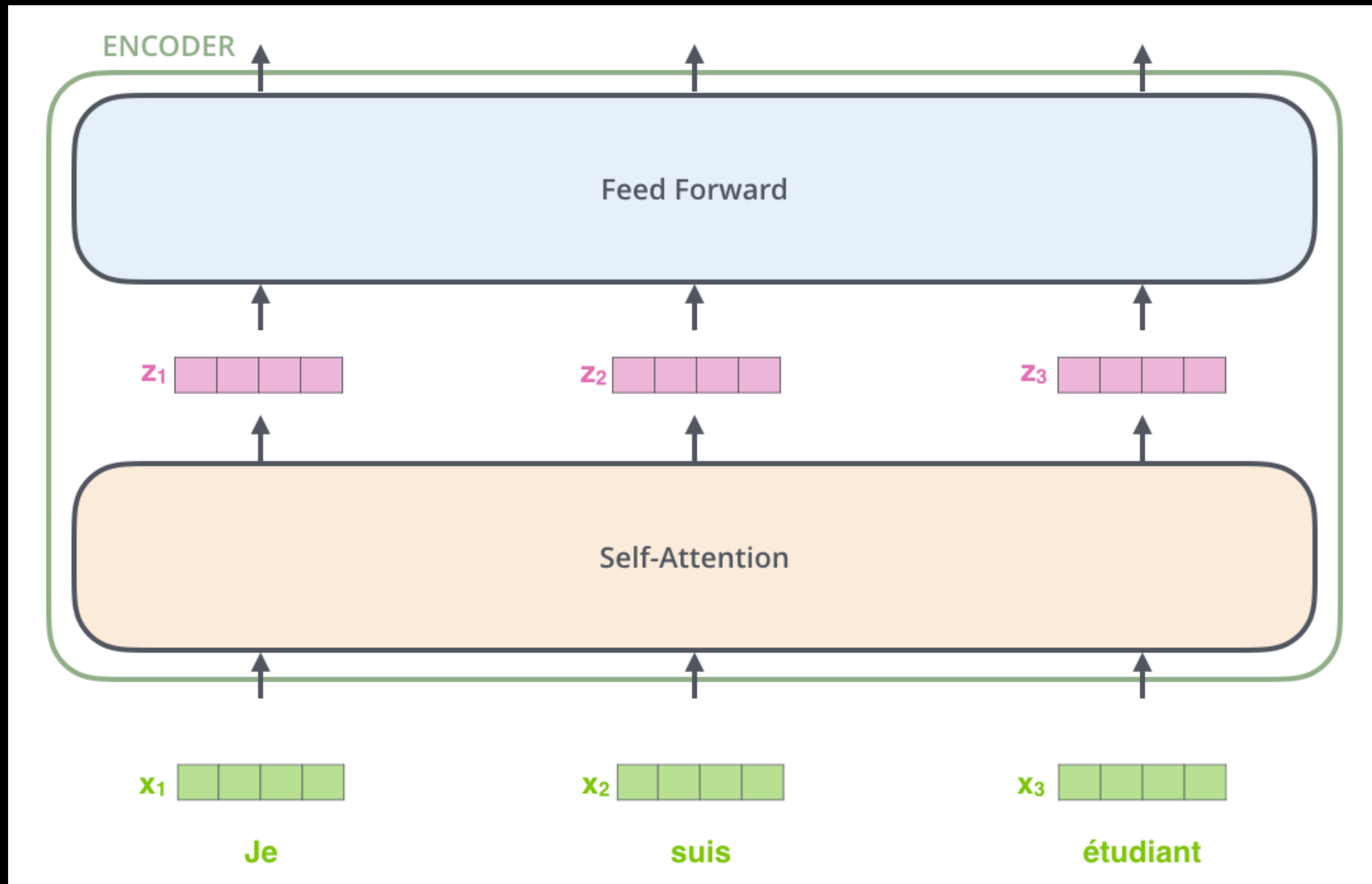
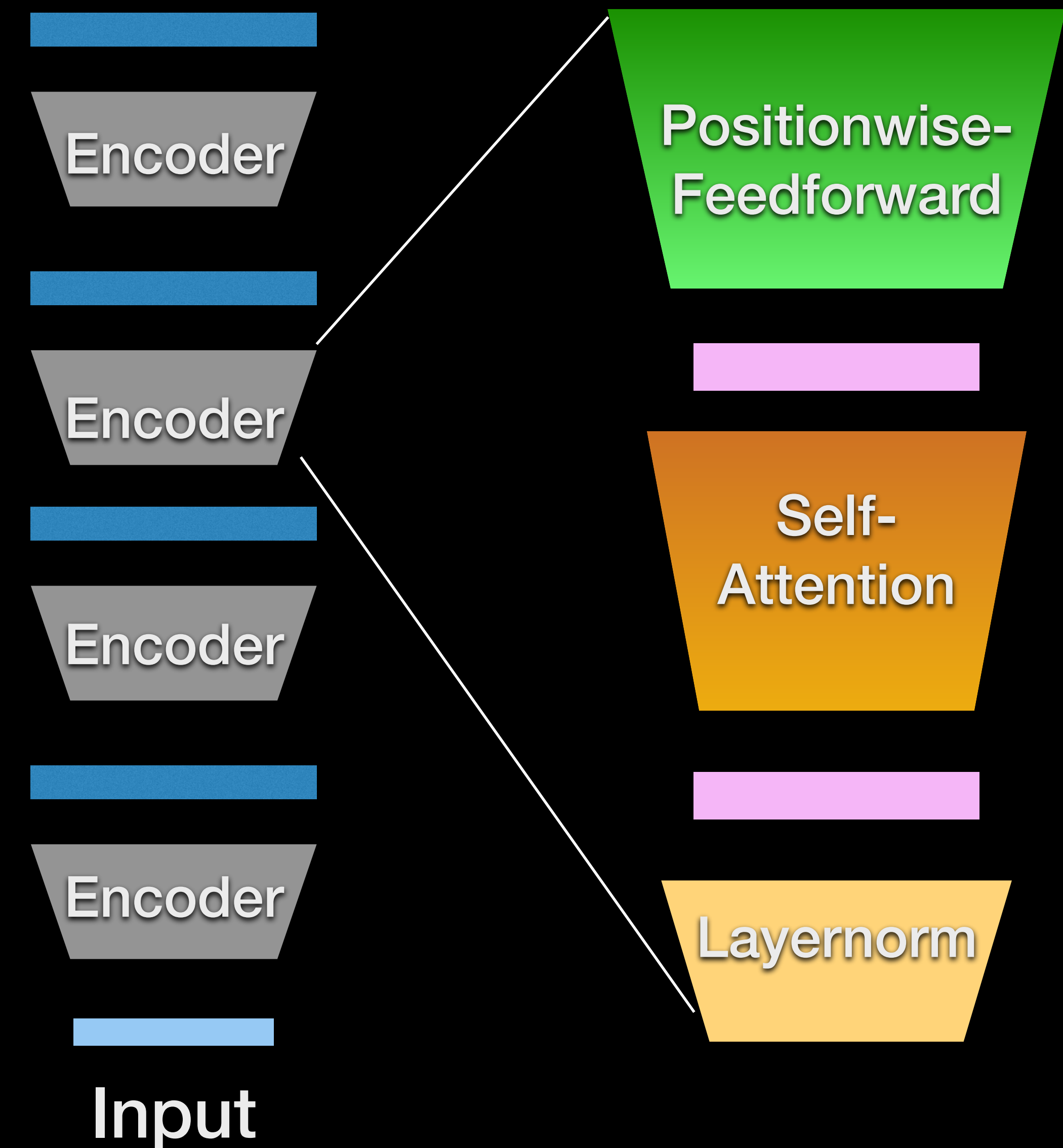
\* Encoder layer output

$$[\mathbf{e}^l(1) \dots \mathbf{e}^l(T)] = \mathbf{E}_{ff}^l \mathbf{W}_{of}^l + \mathbf{1} (\mathbf{b}_{of}^l)^T \in \mathcal{R}^{T \times D}$$

↑ ↑



# Transformers - encoder





# Self Attention - recap

10.1 10.0 9.9  
 1/3 1/3 1/3 → 2/3 (1-2/3)  
 1.01 1.0 0.99

1 2 3 4  
 (The quick brown fox)

\* Illustrative example - The quick brown fox (English) → Der schnelle brane fuchs (German)

	THE	QUICK	BROWN	FOX
THE	0.9	0.1	0	0
QUICK	0.1	0.75	0	0.15
BROWN	0	0	0.7	0.3
FOX	0	0.4	0.35	0.55

T = 4

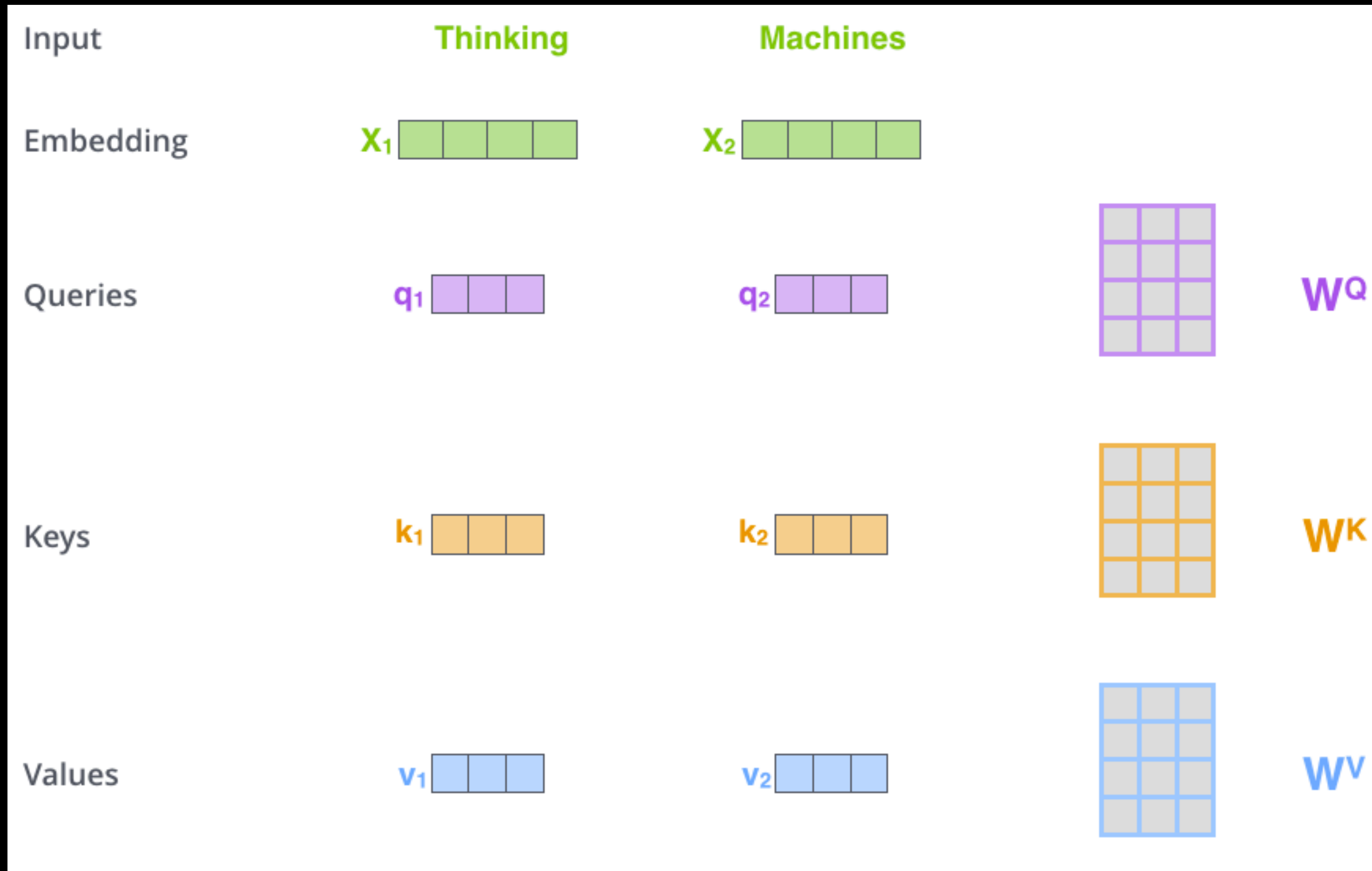
$A_h^l \in \mathbb{R}^{4 \times 4}$   
 ↑ softmax

$$\text{softmax} \left( \frac{QK^T}{\sqrt{d}} \right)$$

→  
 →  
 →  
 →

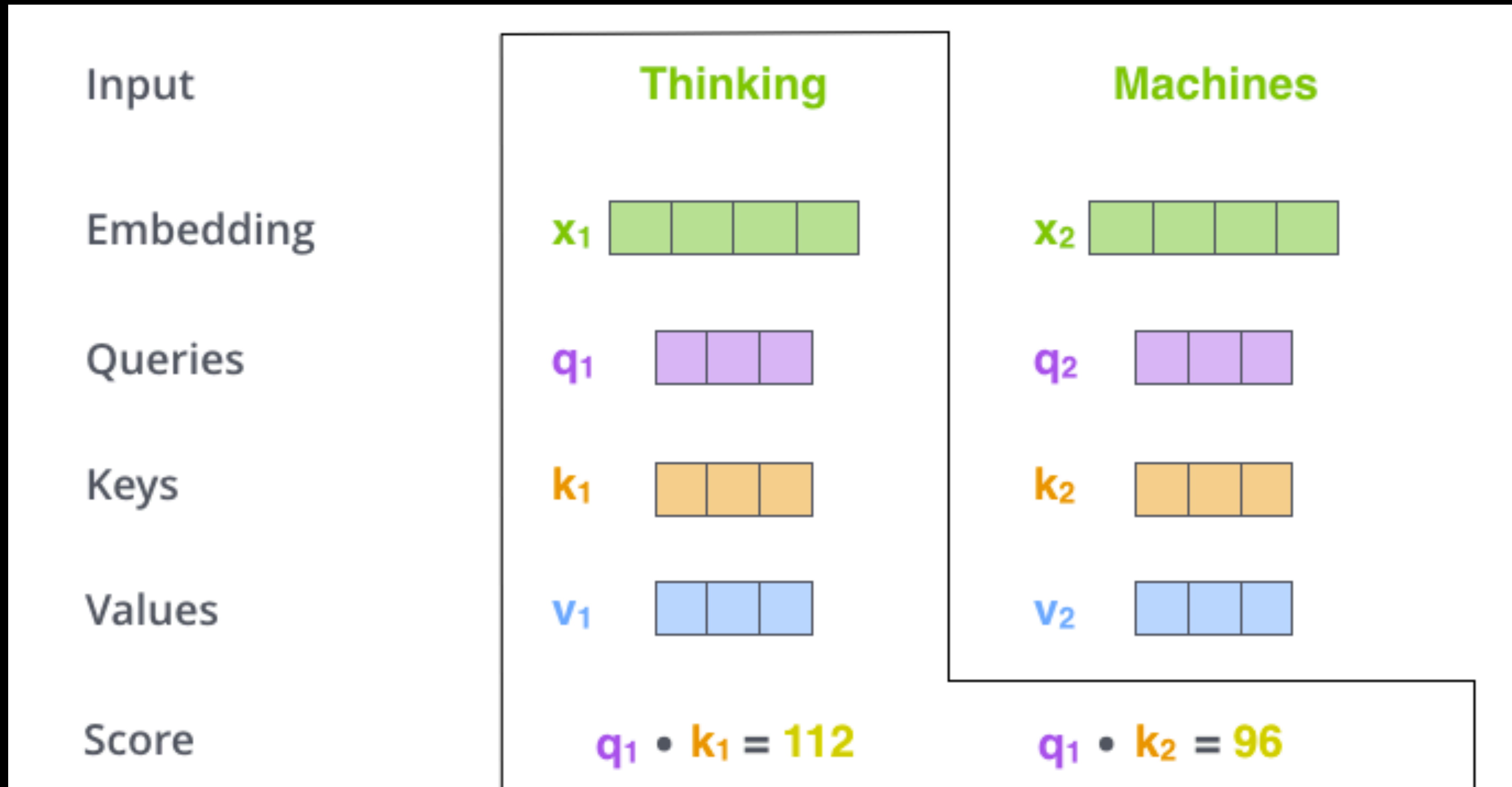


# Self-attention revisited

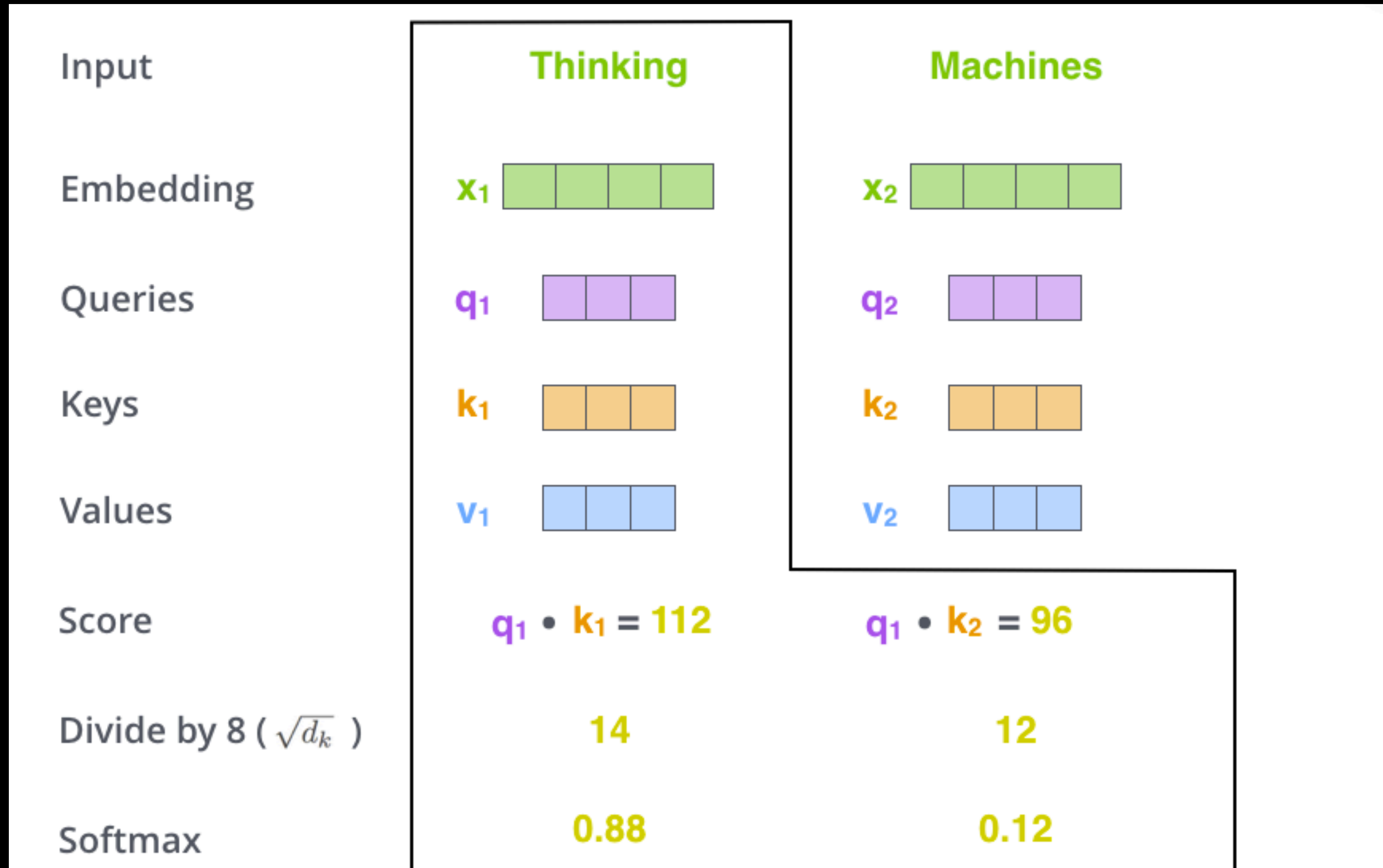




# Self-attention revisited

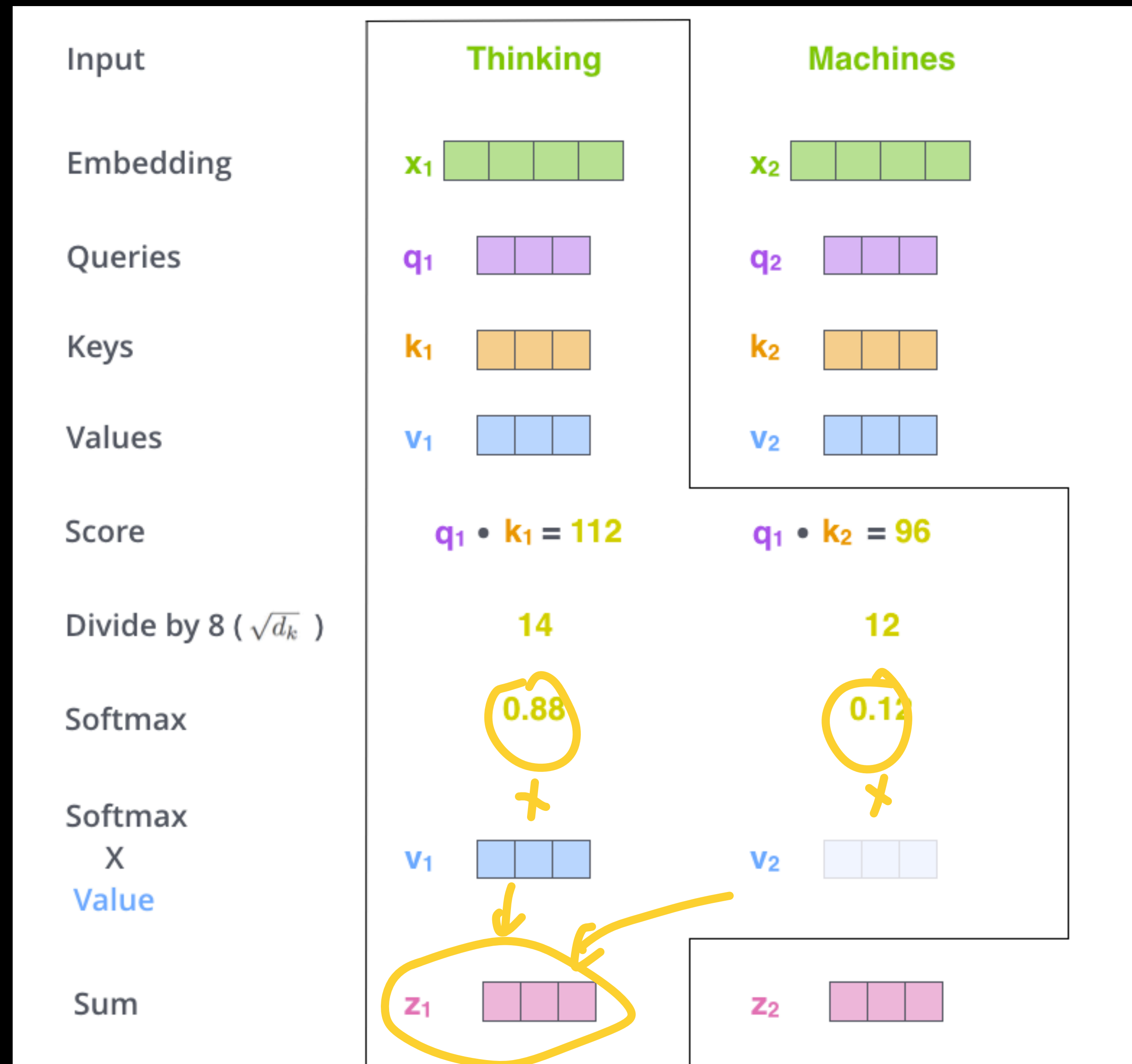


# Self-attention revisited

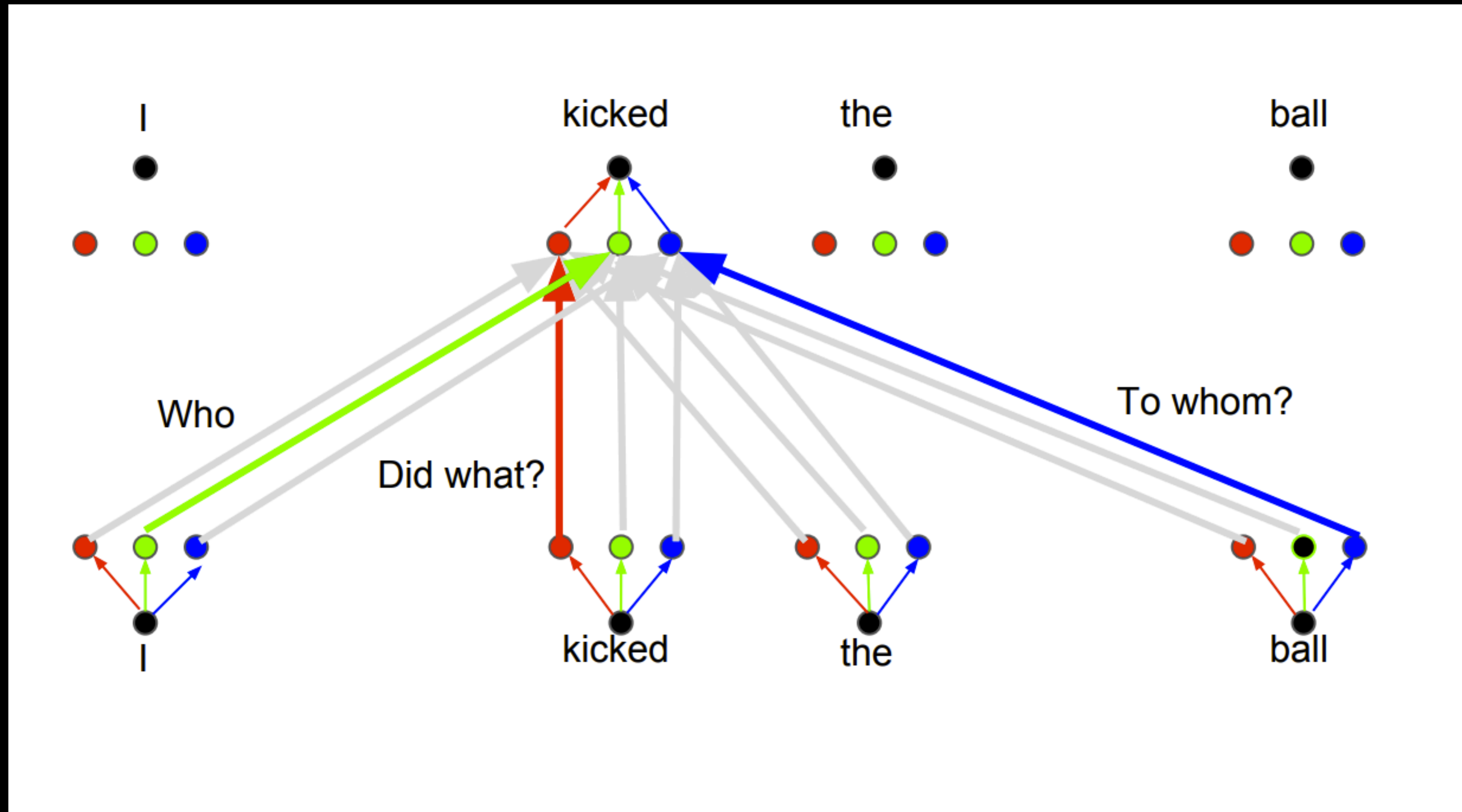




# Self-attention revisited

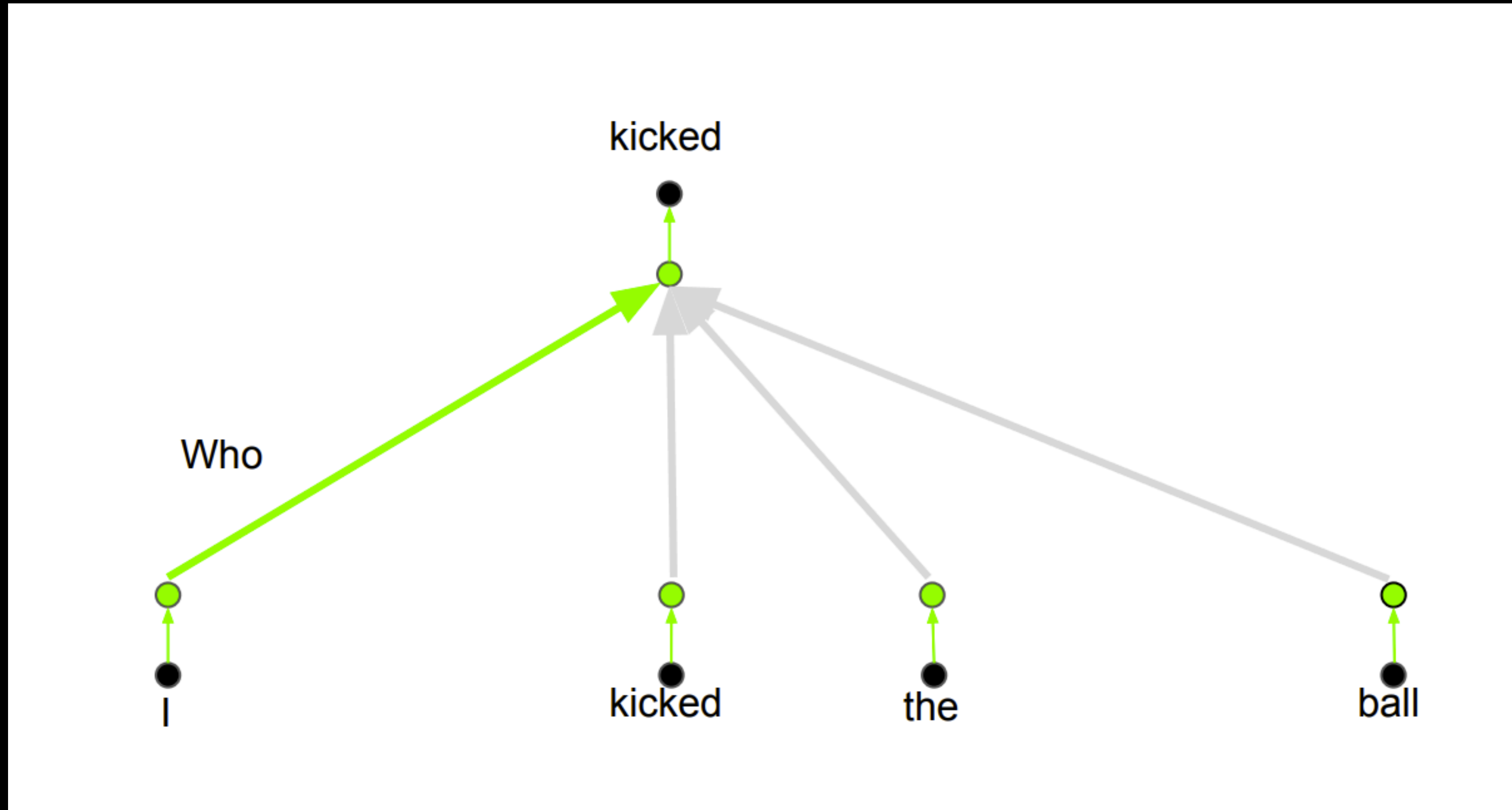


# Self-attention multi-head

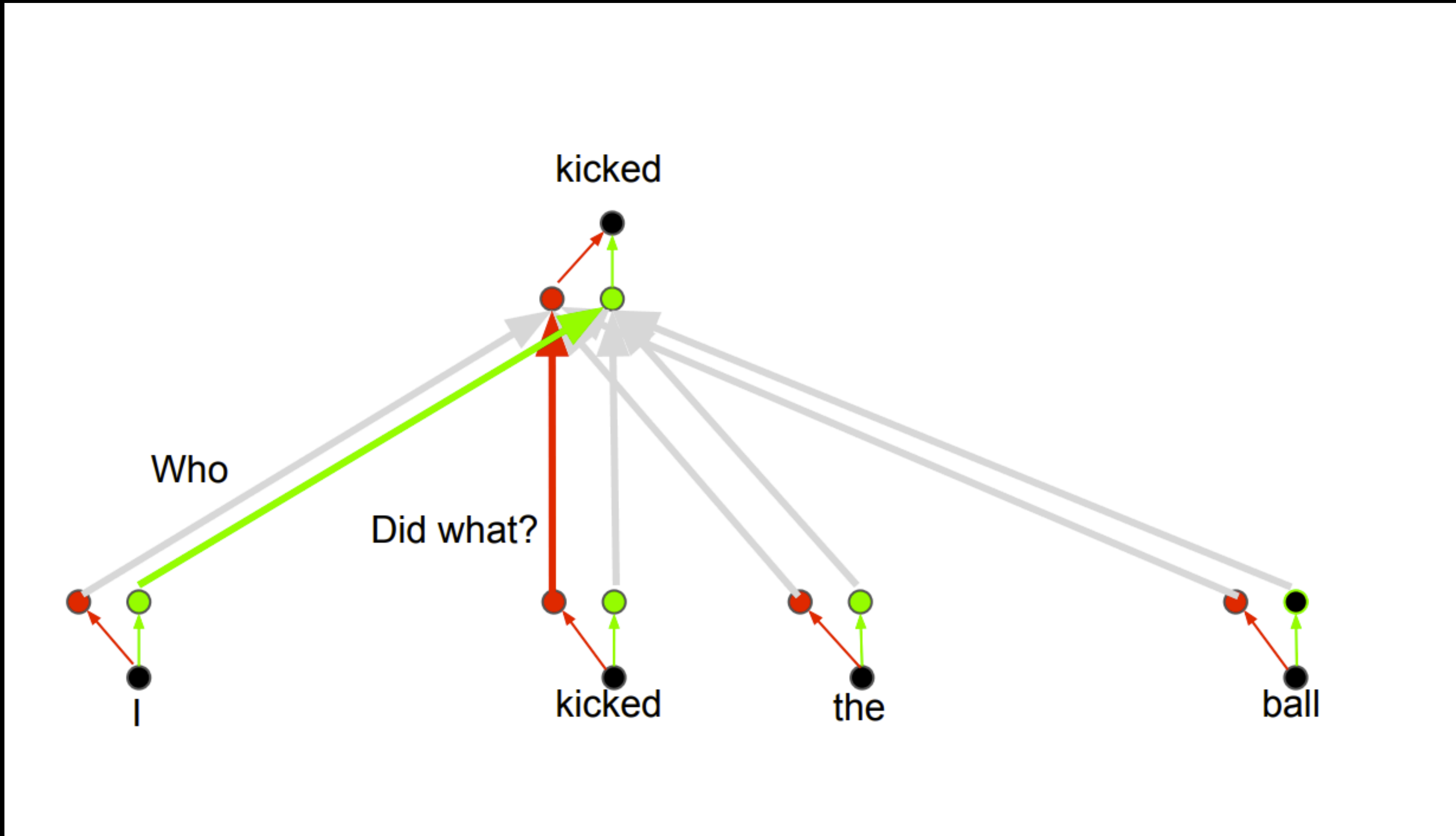




# Self-attention multi-head - role of attention heads

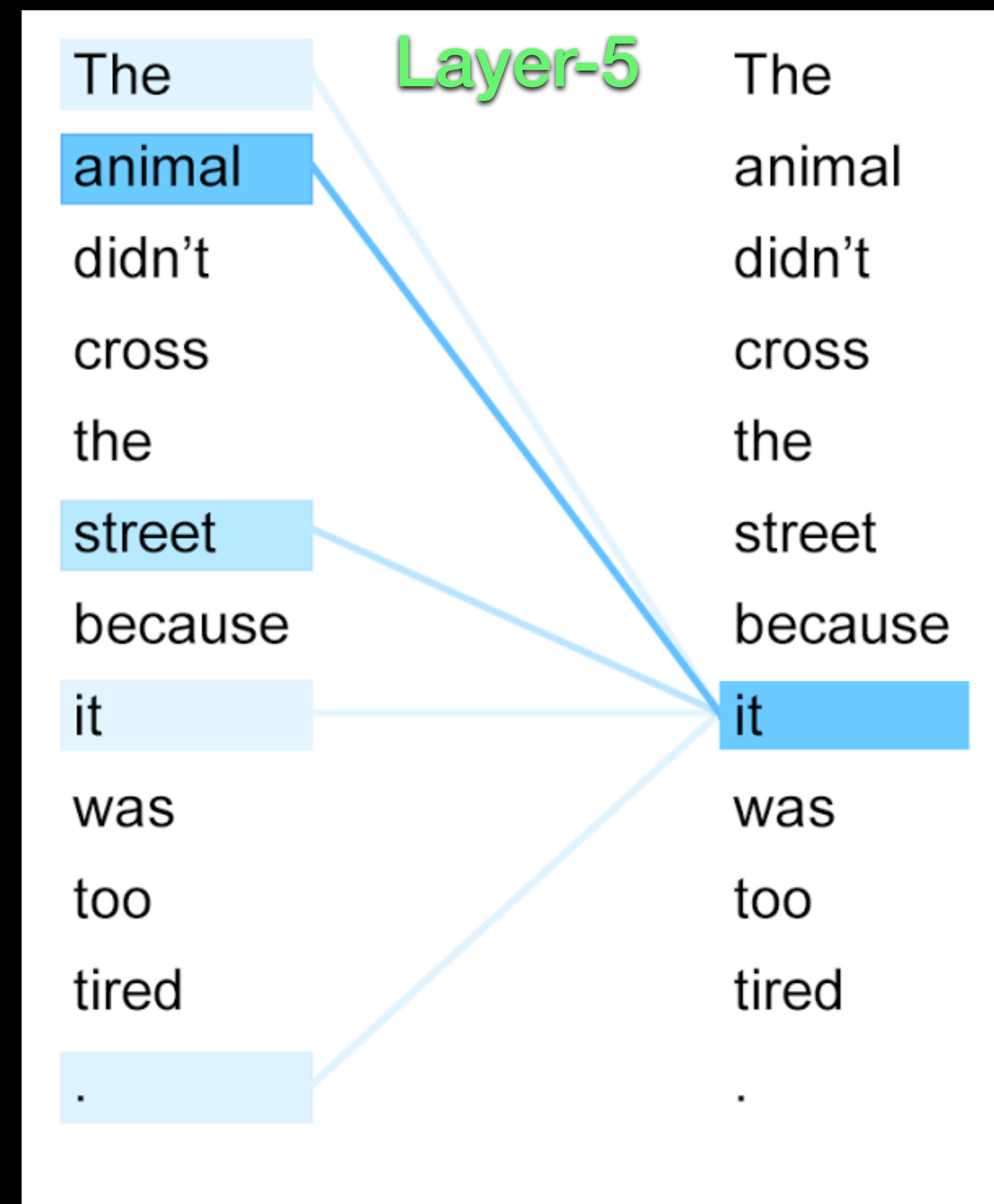


# Self-attention multi-head - role of attention heads

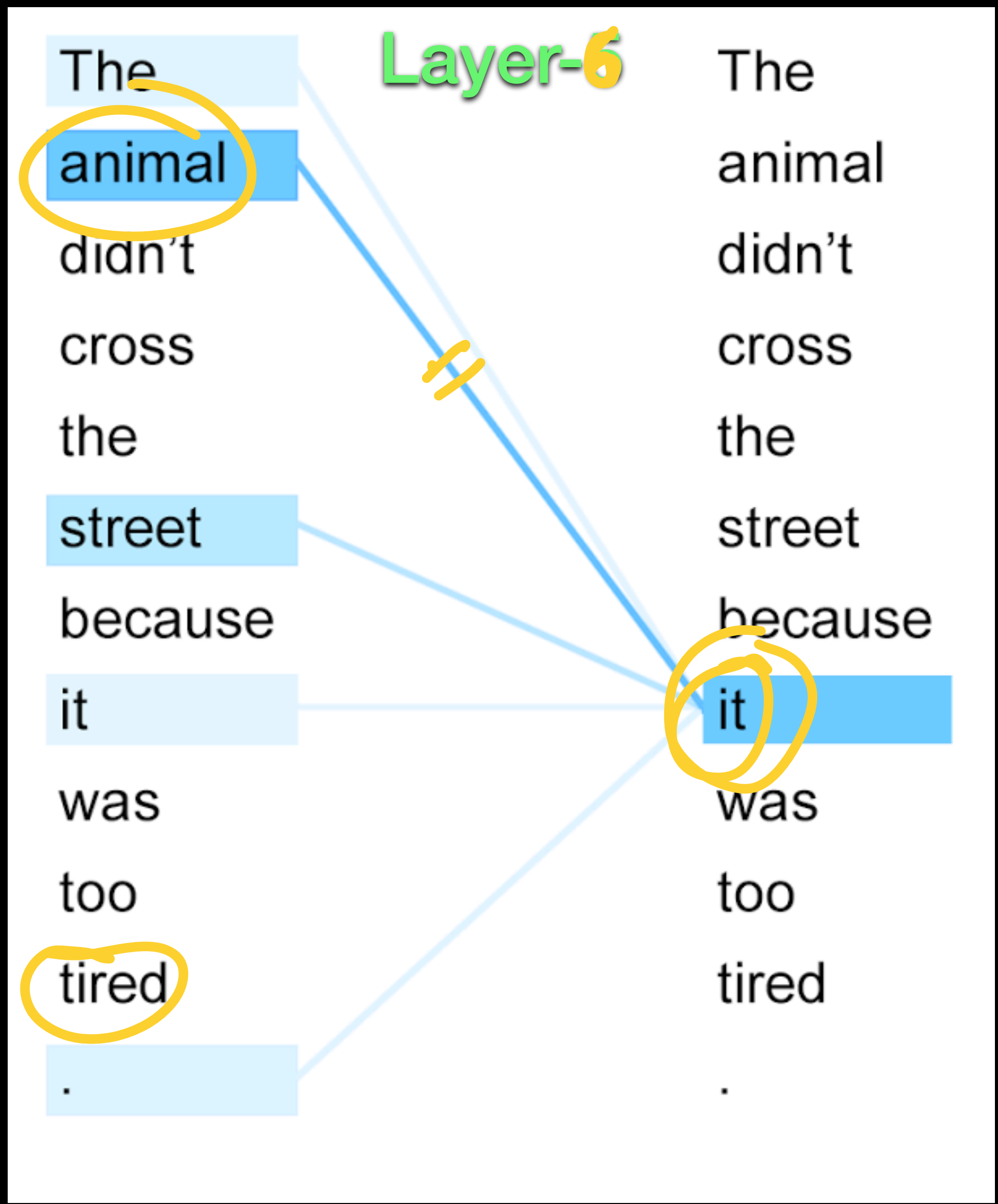
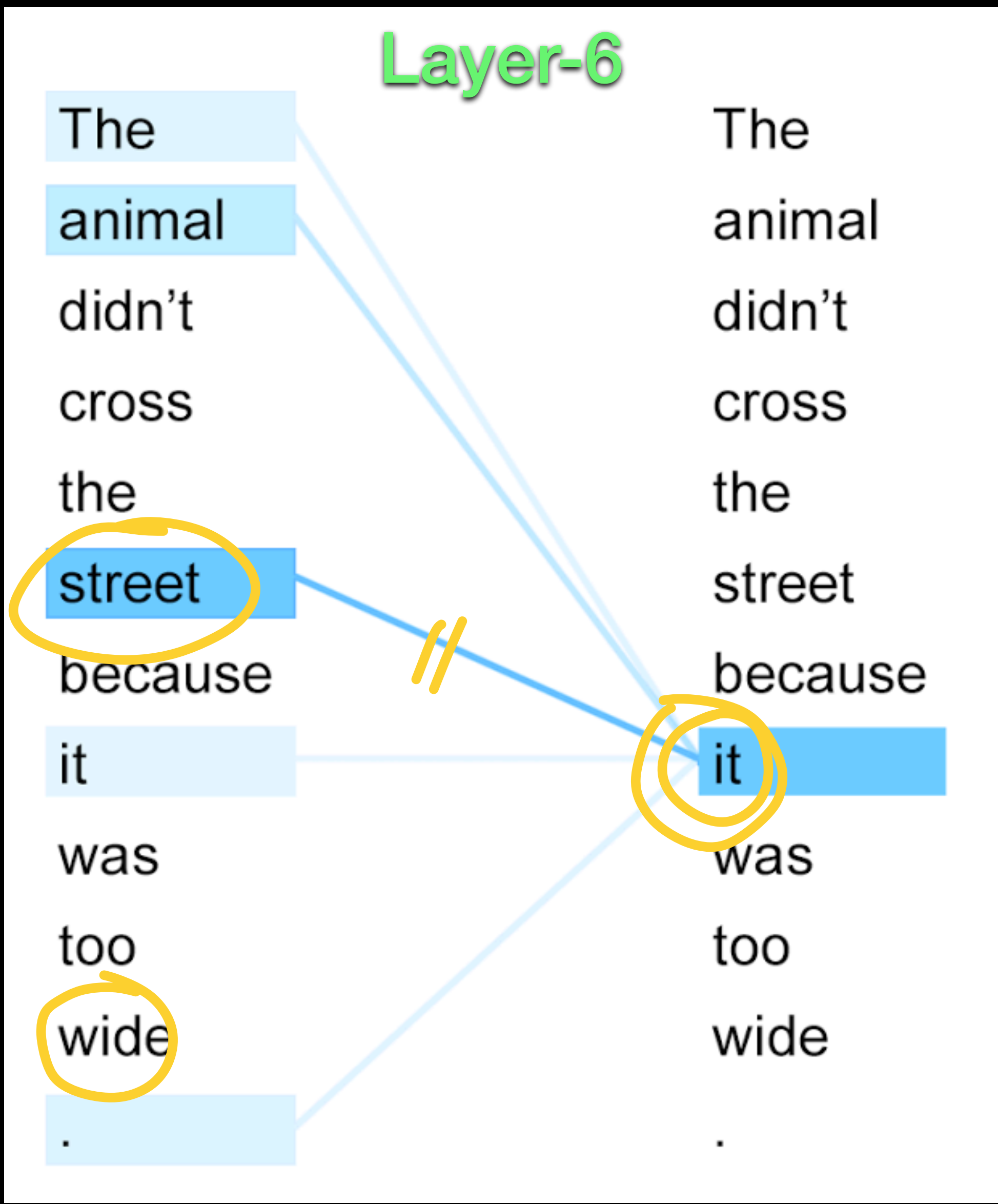




# Self-attention - need for depth



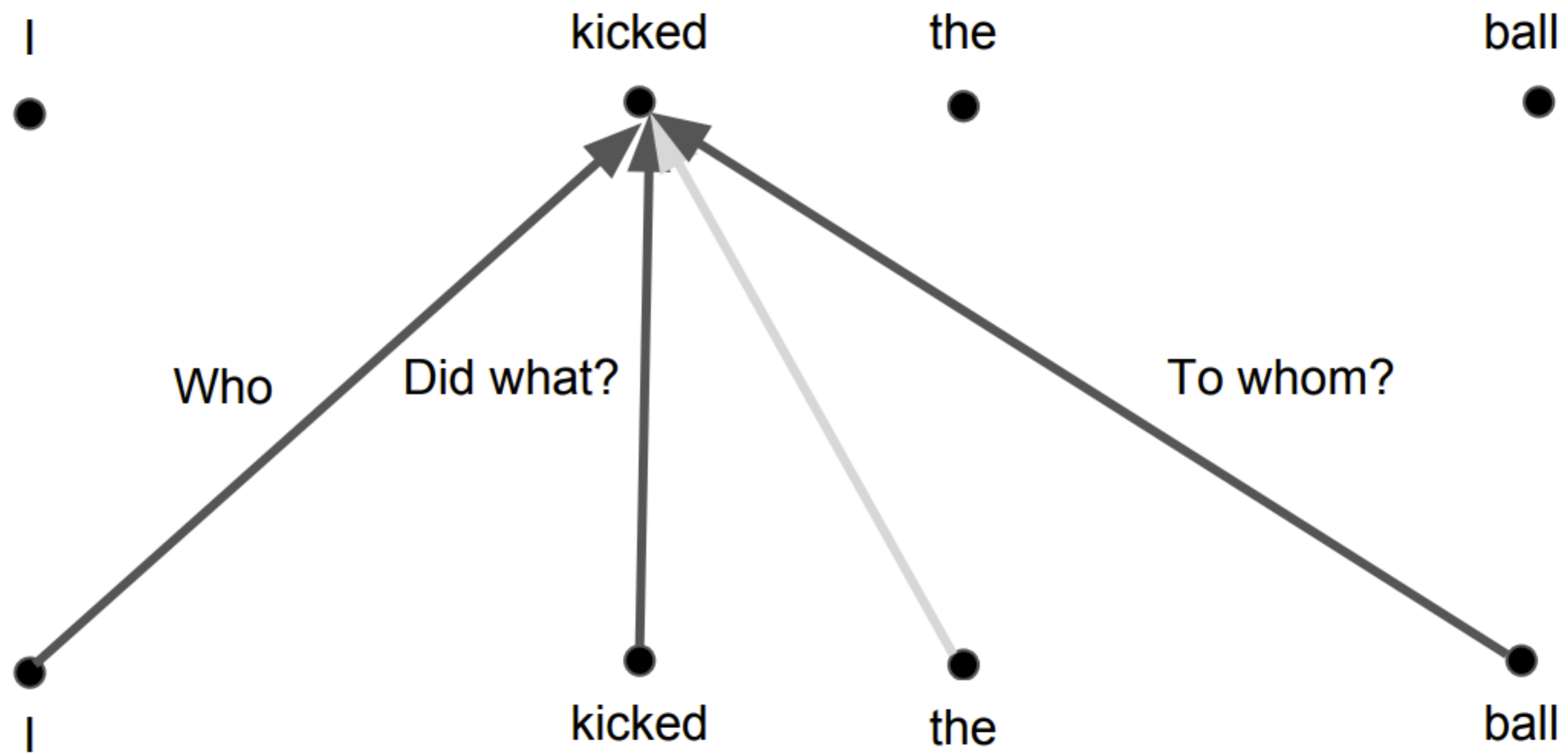
# Self-attention - need for depth





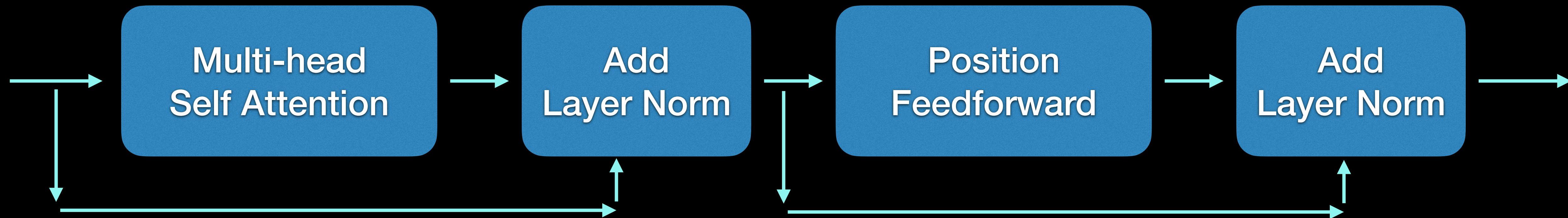
# Need for multi-head attention

## Self-Attention



# Single layer of encoder (typical implementation)

- \* Single encoder layer has typically self-attention skip connection, layer norm and feedforward layer





# Positional encoding

\* No recurrence or position awareness yet in the model

Binary format -

position can encode the rate of change of bits across time

In floating format - one can use sines and cosines

0 :	0	0	0	0	8 :	1	0	0	0
1 :	0	0	0	1	9 :	1	0	0	1
2 :	0	0	1	0	10 :	1	0	1	0
3 :	0	0	1	1	11 :	1	0	1	1
4 :	0	1	0	0	12 :	1	1	0	0
5 :	0	1	0	1	13 :	1	1	0	1
6 :	0	1	1	0	14 :	1	1	1	0
7 :	0	1	1	1	15 :	1	1	1	1



# Positional encoding

\* An example used in the first paper

$$\mathbf{p}(t) \in \mathcal{R}^D$$

$$p_i(t) = \begin{cases} \sin(\omega_k t), & \text{if } i = 2k \\ \cos(\omega_k t), & \text{if } i = 2k + 1 \end{cases} \quad k \in \left\{1 \dots \frac{D}{2}\right\}$$

$$\omega_k = \frac{1}{10000^{\frac{2k}{D}}}$$

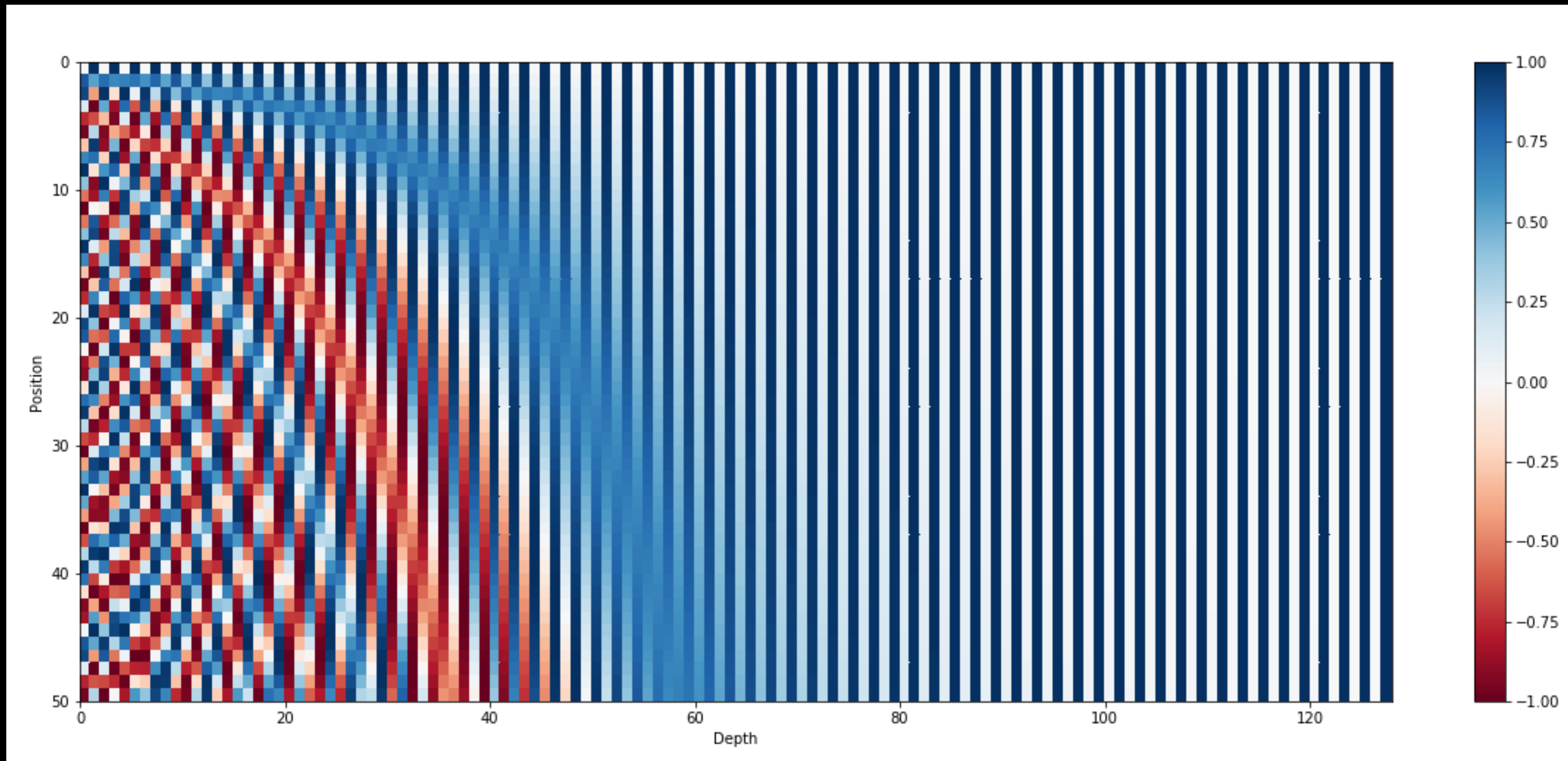
$$\mathbf{x}(t) = \mathbf{x}(t) + \mathbf{p}(t)$$





# Positional encoding

\* An example used in the first paper  $\mathbf{p}(t) \in \mathcal{R}^D$  [T=50, D=128]



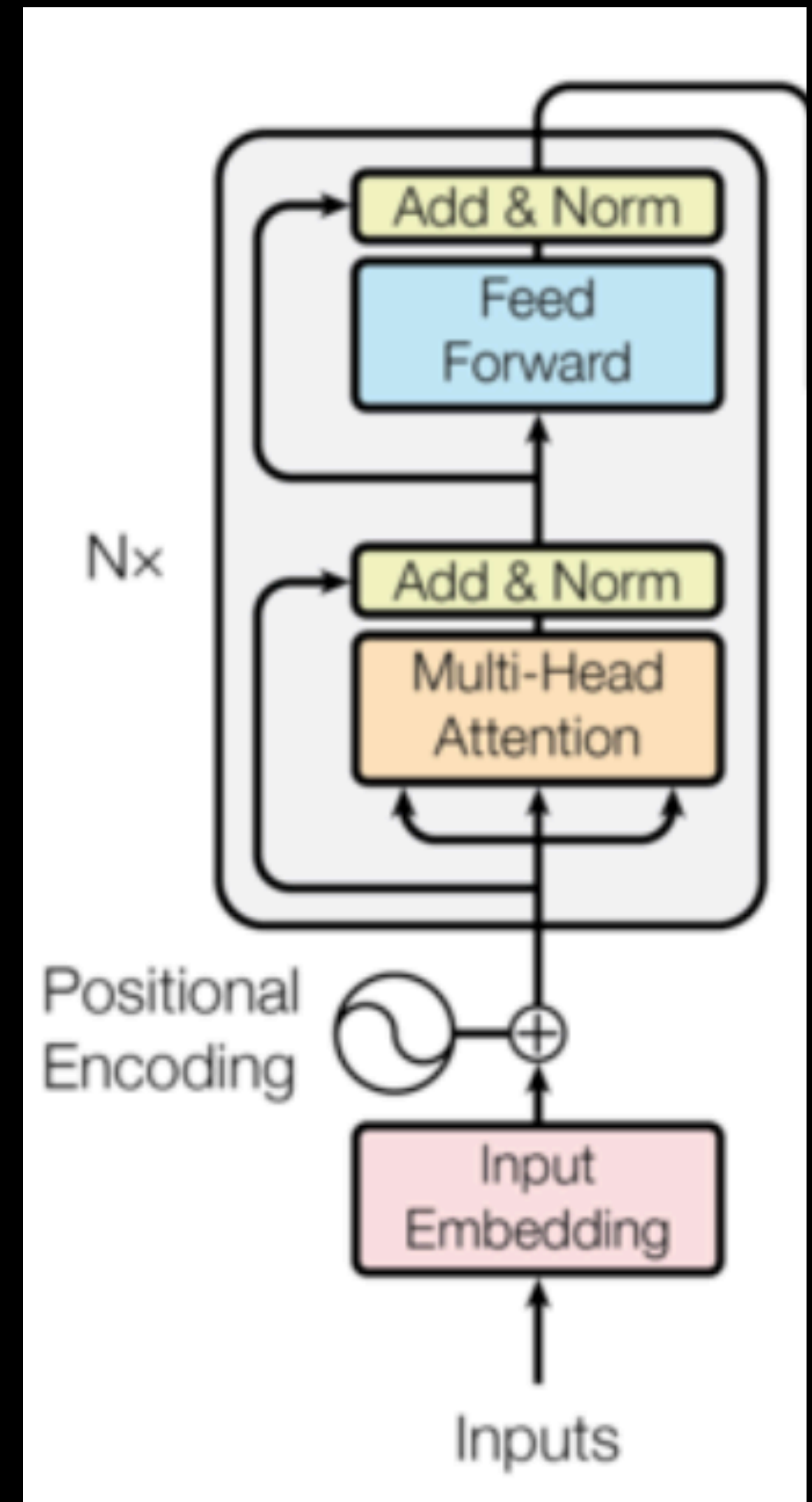
T

D



# Transformer encoder - overview

Reading Assignment - "Attention is All You Need"  
<https://arxiv.org/pdf/1706.03762.pdf>



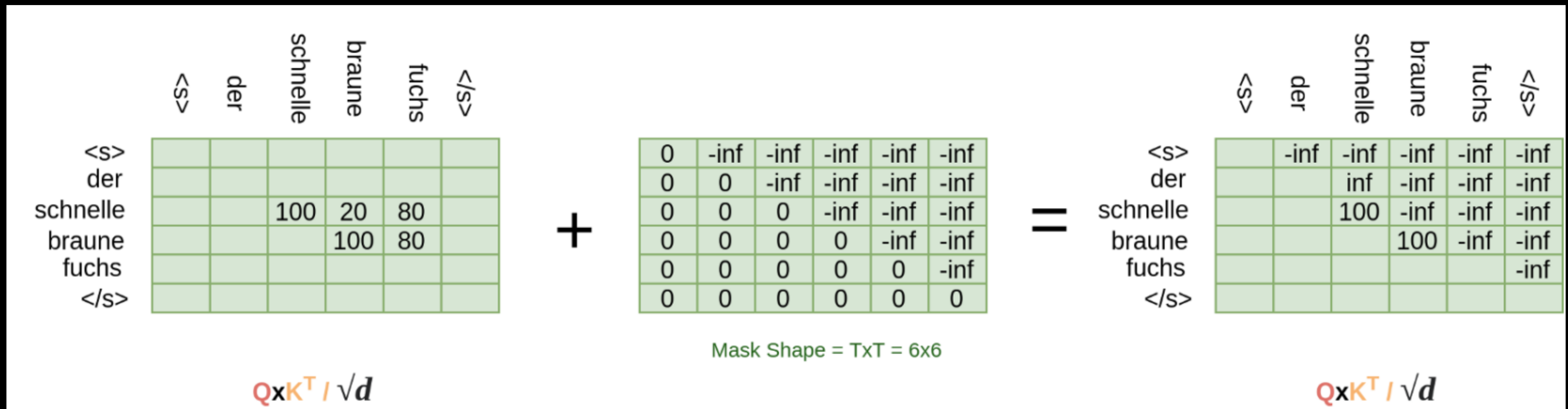


# Transformer decoder

## \* Masked self-attention layer -

✓ Mask makes the output dependencies causal

★ Only the past is used to encode the attention.



# Transformer decoder

## \* Masked self-attention layer -

✓ Mask makes the output dependencies causal

★ Only the past is used to encode the attention.

$$\text{Softmax}\left\{\frac{\mathbf{QK}^T}{\sqrt{d}}\right\} \mathbf{V} \longrightarrow \text{Softmax}\left\{\text{Mask} + \frac{\mathbf{QK}^T}{\sqrt{d}}\right\} \mathbf{V}$$

★ Make the attention matrix to be lower triangular





# Transformer decoder

## \* Masked self-attention layer -

✓ Mask makes the output dependencies causal

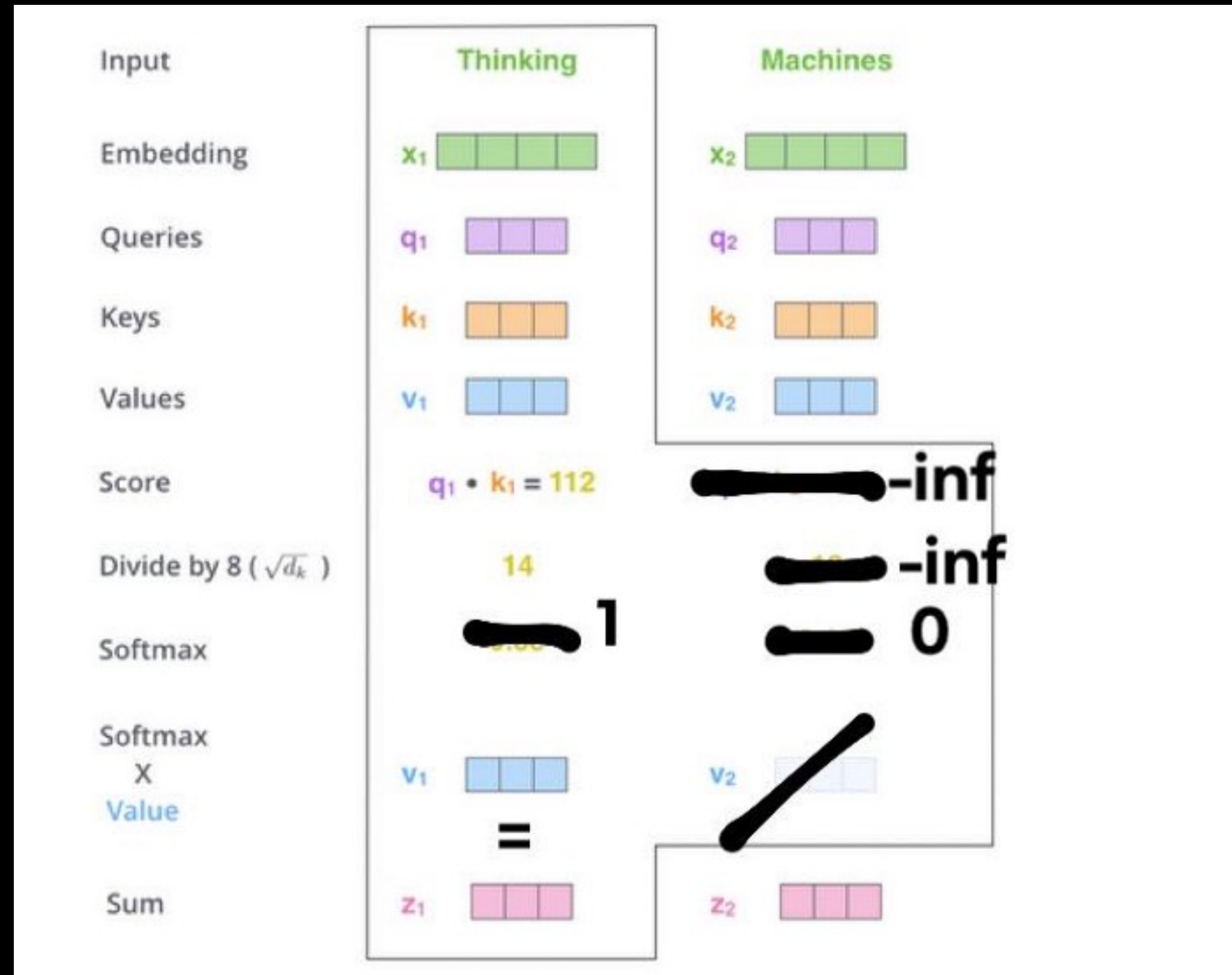
★ Only the past is used to encode the attention.

	<s>	der	schnelle	braune	fuchs	</s>
<s>		0	0	0	0	0
der			0	0	0	0
schnelle			.75	0	0	0
braune				.85	0	0
fuchs						0
</s>						

$\text{softmax}(QxK^T / \sqrt{d})$



# Transformer decoder





# Encoder-decoder attention

✳ Use the Key and Value matrices from the last layer of the encoder

$$\mathbf{Q}_h^p = \overline{\mathbf{D}}^{p-1} \mathbf{W}_h^{p,Q} + \mathbf{1}(\mathbf{b}_h^{p,Q})^T \in \mathcal{R}^{S \times d}$$

$$\mathbf{K}_h^p = \mathbf{E}^L \mathbf{W}_h^{p,K} + \mathbf{1}(\mathbf{b}_h^{p,K})^T \in \mathcal{R}^{T \times d}$$

$$\mathbf{V}_h^p = \mathbf{E}^L \mathbf{W}_h^{p,V} + \mathbf{1}(\mathbf{b}_h^{p,V})^T \in \mathcal{R}^{T \times d}$$

$$\mathbf{D}_h^p = \text{softmax}\left(\frac{\mathbf{Q}_h^p (\mathbf{K}_h^p)^T}{\sqrt{d}}\right) \mathbf{V}_h^p \in \mathcal{R}^{S \times d}$$

$$h = \{1..H\} \quad \text{heads} \quad d = \frac{D}{H}$$



# Transformer - decoder

## \* Decoder Layer Output

\* 
$$[\mathbf{d}^p(1) \dots \mathbf{d}^p(S)] = \text{ReLU} \left( \mathbf{D}_{ff}^p \mathbf{W}_{of}^p + \mathbf{1} (\mathbf{b}_{of}^p)^T \right) \in \mathcal{R}^{S \times D}$$



# Transformer - full pipeline

Reading Assignment - "Attention is All You Need"  
<https://arxiv.org/pdf/1706.03762.pdf>

