

Housekeeping

- * Attendance
 - ✓ We will use the recorded sessions for attendance
 - * If you are unable to attend live sessions (due to network or other issues, please indicate by email before or after class to the instructor and copy the FAs).
- * Mid-term exam
 - → 1st week of Dec. (Modules I and II).



Housekeeping

- * lst mini-project
 - √ Deadlines
 - * Abstract submission deadline (Nov 2nd, Monday)
 - * Using the google form given in the webpage
 - * Solo projects or 2-member projects
 - * Indicate roles of each member in 2-member project
 - ★ 200 page abstract of the work. If modifications are needed, we will review and let you know in 2-3 days.



Housekeeping

- * 1st mini-project
 - √ Deadlines
 - * Report and presentation slides (Nov 18th, 10 AM).
 - ★ 1-page pdf with second page only for references and tools used (Template will be provided).
 - * Report Indicate prior work, technical details and your contribution. Strictly adhere to the guidelines given in the template.
 - ★ Slides (max 4 slides) 4 min presentation for solo project and 6 min. for two member teams. 3 mins for your presentation and 1 min for Q&A.
 - * Two slots are available on 2 days (pick the suitable based on your other class schedules).



Recap of previous class



State of affairs

* Encoder-decoder models with attention.

→ self attention and multi-head attention

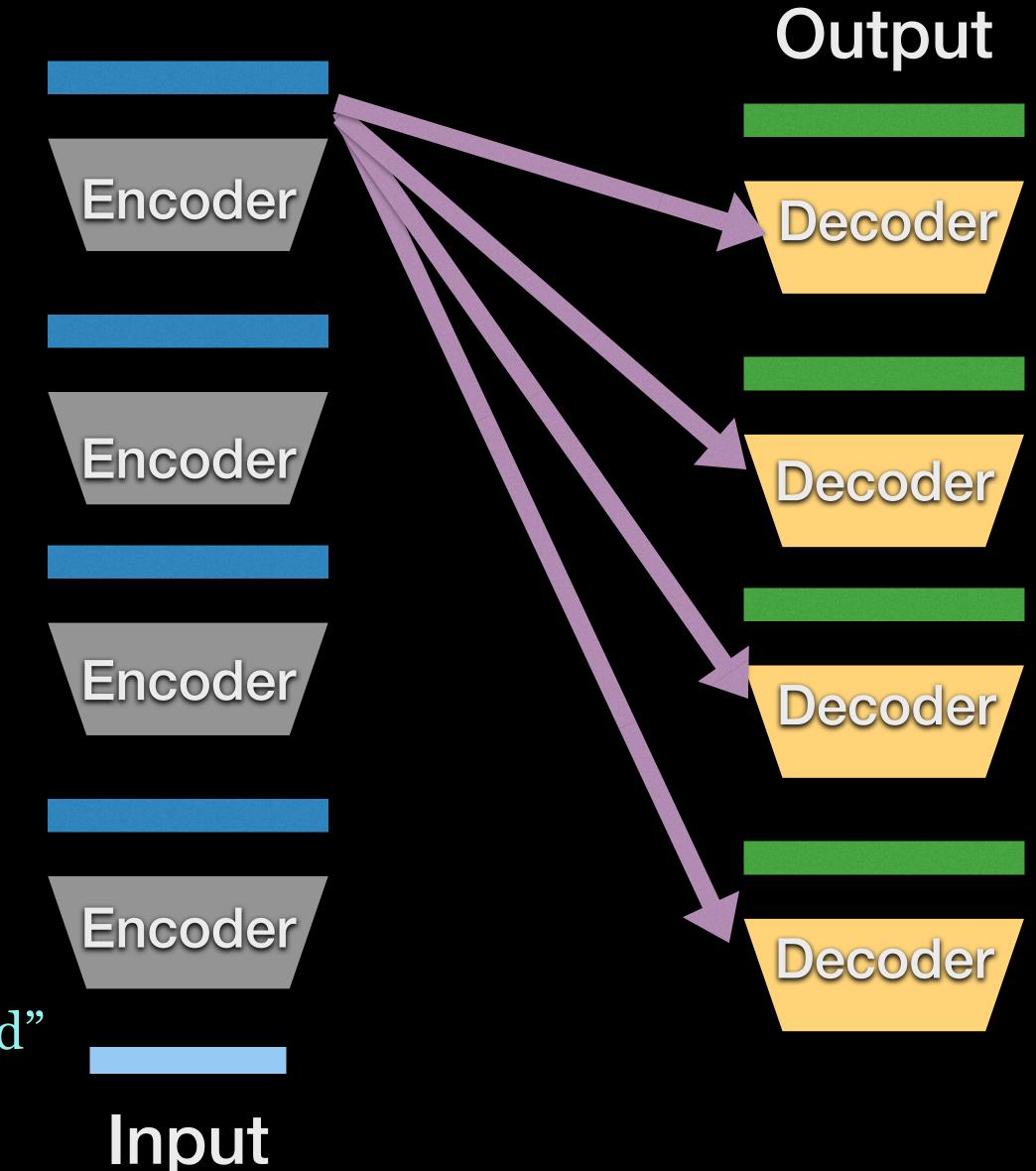
* Transformer models - Introduction



Transformers

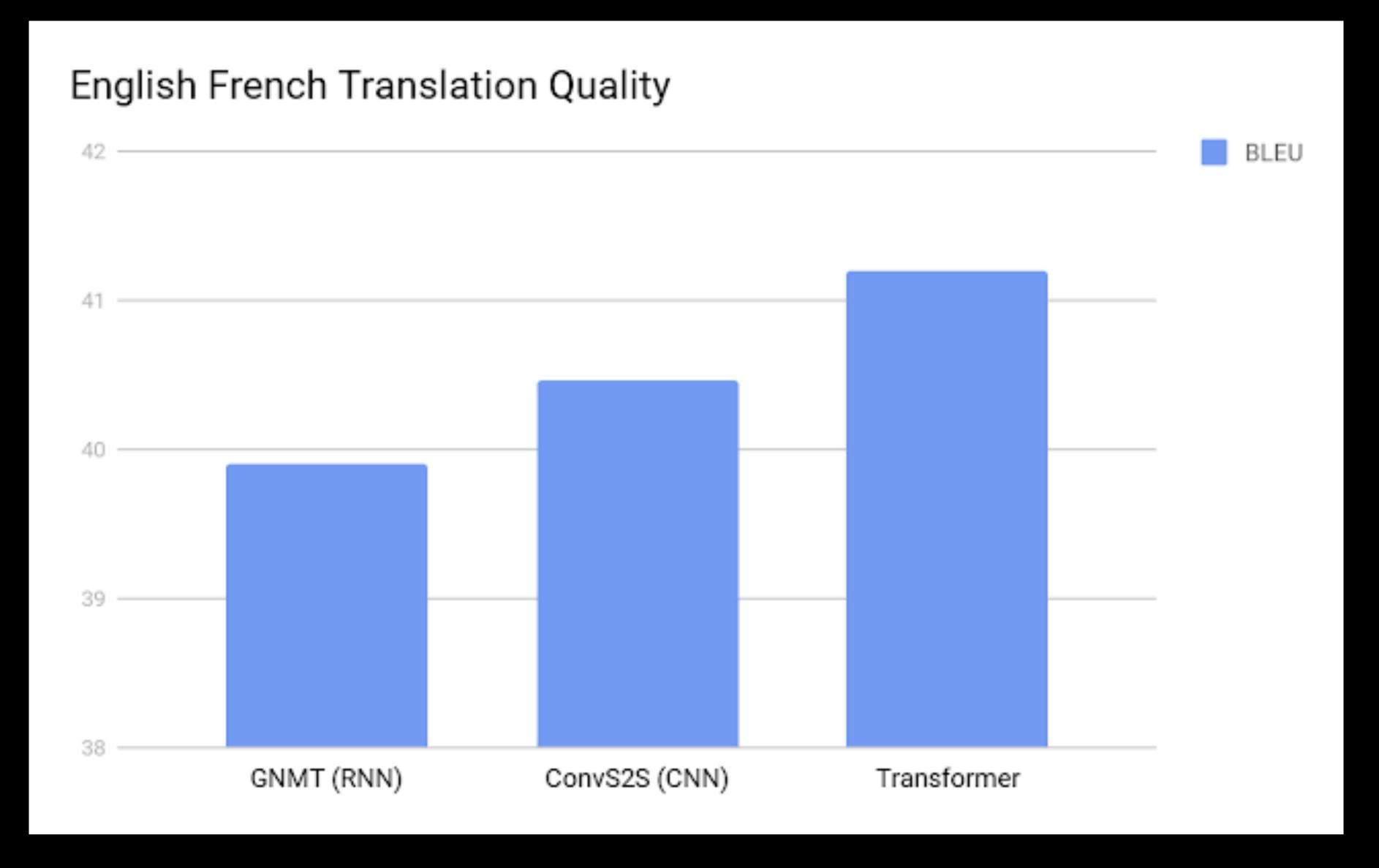
- * Encoder Decoder architecture based models.
- * Uses only feed forward architectures with self-attention.
 - → Multi-head self attention.
- * All the encoder layers and the decoder layers have the same set of operations.

Reading Assignment - "Attention is All You Need" https://arxiv.org/pdf/1706.03762.pdf





Transformers - the state of art in NMT



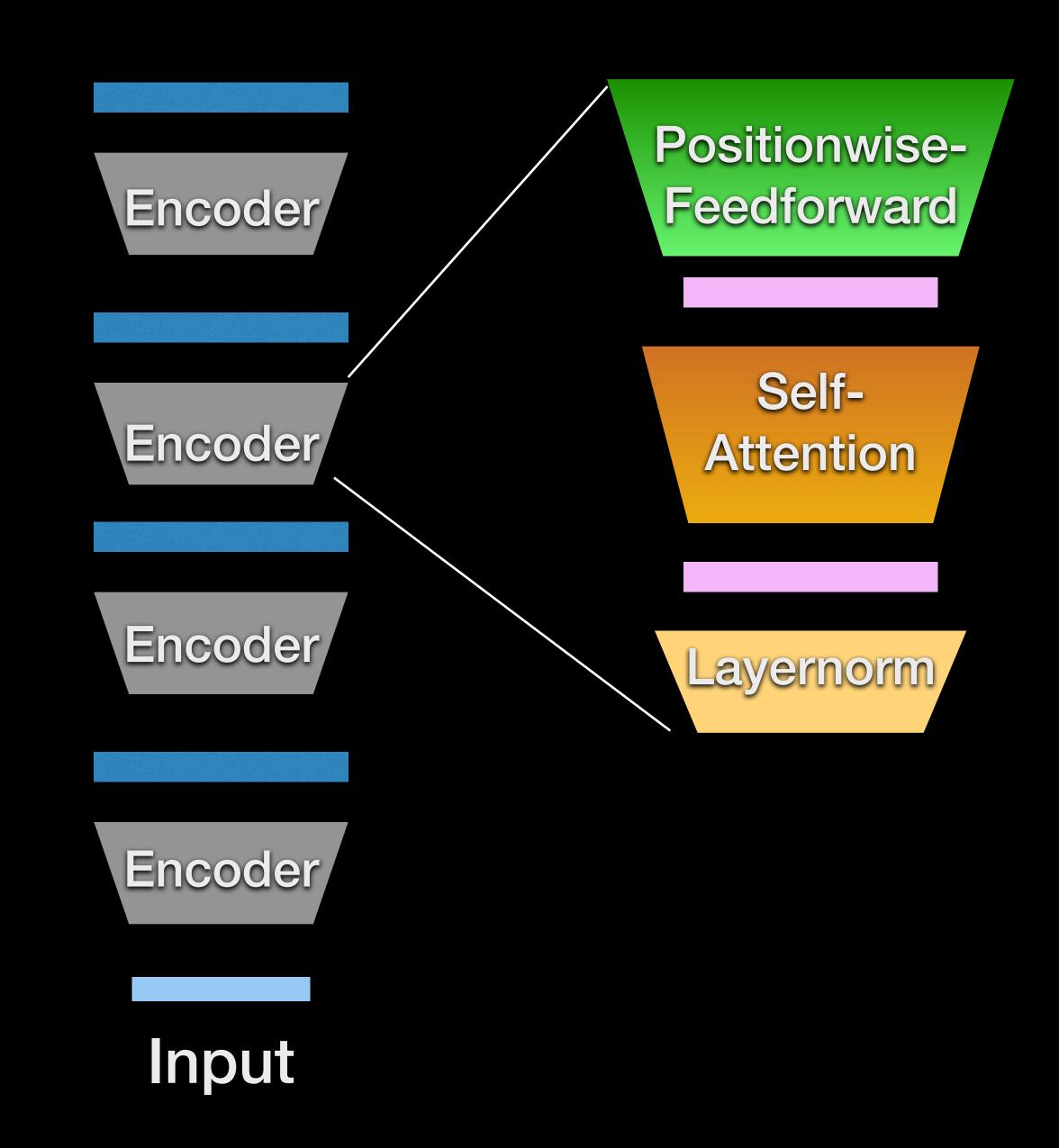


Transformers - the state of art in NMT



Transformers

- * Encoder layers
 - → Consist of layer norm
 - → Self attention (multi-head)
 - → Positionwise feedforward
 - ✓ May also consist of skip connections.





* Let $\mathbf{x}(1)...\mathbf{x}(T)$ denote the input and let $\mathbf{e}^l(1)...\mathbf{e}^l(T)$ denote encoder outputs at layer 1.

$$\overline{\mathbf{E}}^{l-1} = Layernorm([\mathbf{e}^{l-1}(1)...\mathbf{e}^{l-1}(T)]^T) \in \mathcal{R}^{T \times D}$$

* Definition of layer norm

$$Layernorm(\mathbf{e}^{l}(t)) = \frac{\boldsymbol{\alpha}^{l}}{\boldsymbol{\sigma}_{\mathbf{e}^{l}(t)}} \odot (\mathbf{e}^{l}(t) - \boldsymbol{\mu}_{\mathbf{e}^{l}(t)}) + \boldsymbol{\beta}^{l}$$



* Querry, Key and Value

$$\mathbf{Q}_h^l = \overline{\mathbf{E}}^{l-1} \mathbf{W}_h^{l,Q} + \mathbf{1}(\mathbf{b}_h^{l,Q})^T \in \mathcal{R}^{T \times d}$$
 $\mathbf{K}_h^l = \overline{\mathbf{E}}^{l-1} \mathbf{W}_h^{l,K} + \mathbf{1}(\mathbf{b}_h^{l,K})^T \in \mathcal{R}^{T \times d}$
 $\mathbf{V}_h^l = \overline{\mathbf{E}}^{l-1} \mathbf{W}_h^{l,V} + \mathbf{1}(\mathbf{b}_h^{l,V})^T \in \mathcal{R}^{T \times d}$

*
$$\mathbf{W}_h^{l,Q}, \mathbf{W}_h^{l,K}, \mathbf{W}_h^{l,V} \in \mathcal{R}^{D \times d}$$
 $\mathbf{b}_h^{l,Q}, \mathbf{b}_h^{l,K}, \mathbf{b}_h^{l,V} \in \mathcal{R}^{d \times 1}$

$$h = \{1..H\}$$
 heads $d = \frac{D}{H}$ $1 \in \mathcal{R}^{T imes 1}$ all ones



* Multi-head attention

$$\hat{\mathbf{A}}_h^l = \mathbf{Q}_h^l(\mathbf{K}_h^l)^T \in \mathcal{R}^{T \times T}$$

$$\hat{\mathbf{A}}_h^l = softmax(\frac{\hat{\mathbf{A}}_h^l}{\sqrt{d}})$$

$$\mathbf{C}_h^l = \mathbf{A}_h^l \mathbf{V}_h^l \in \mathcal{R}^{T \times D}$$

* Context vector from self-attention

$$\mathbf{C}^l = [\mathbf{C}_1^1...\mathbf{C}_H^l] \in \mathcal{R}^{T \times D}$$



* Position wise feedforward layer

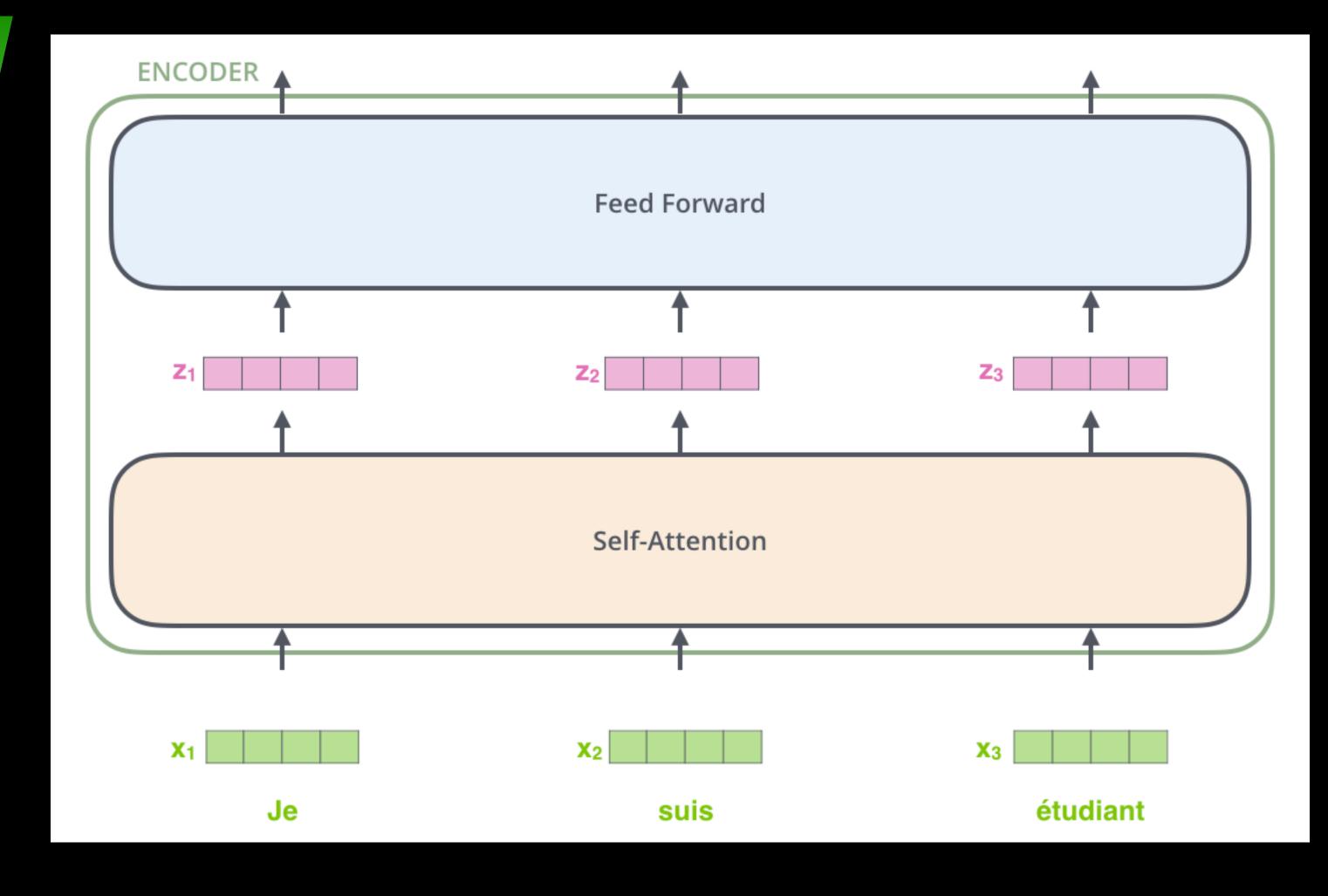
$$\mathbf{E}_{ff}^{l} = ReLU(\mathbf{C}^{l}\mathbf{W}_{ff}^{l} + \mathbf{1b}_{ff}^{T}) \in \mathcal{R}^{T \times d_{ff}}$$

* Encoder layer output

$$[\mathbf{e}^l(1)...\mathbf{e}^l(T)] = \mathbf{E}_{ff}^l \mathbf{W}_{of}^l + \mathbf{1}(\mathbf{b}_{of}^l)^T \in \mathcal{R}^{T \times D}$$



Positionwise-Encoder Feedforward Encoder Self-Attention Encoder Encoder ayernorm







Self Attention - recap

* Illustrative example - The quick brown fox (English) —> Der shnelle brane

fuchs (German)

	THE	QUICK	BROWN	FOX
THE	0.9	0.1	0	0
QUICK	0.1	0.75	0	0.15
BROWN	0	0	0.7	0.3
FOX	0	0.2	0.35	0.55



Input	Thinking	Machines	
Embedding	X ₁	X ₂	
Queries	q ₁	q ₂	WQ
Keys	k ₁	k ₂	WK
Values	V ₁	V ₂	W۷



Input

Embedding

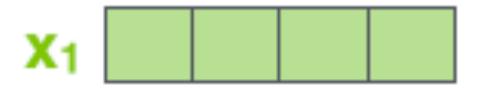
Queries

Keys

Values

Score

Thinking



q₁

k₁

V₁

 $q_1 \cdot k_1 = 112$

Machines

X₂

q₂

K₂

V₂

 $q_1 \cdot k_2 = 96$



Input

Embedding

Queries

Keys

Values

Score

Divide by 8 ($\sqrt{d_k}$)

Softmax

Thinking



q₁

(1

V₁

 $q_1 \cdot k_1 = 112$

14

0.88

Machines

X₂

q₂

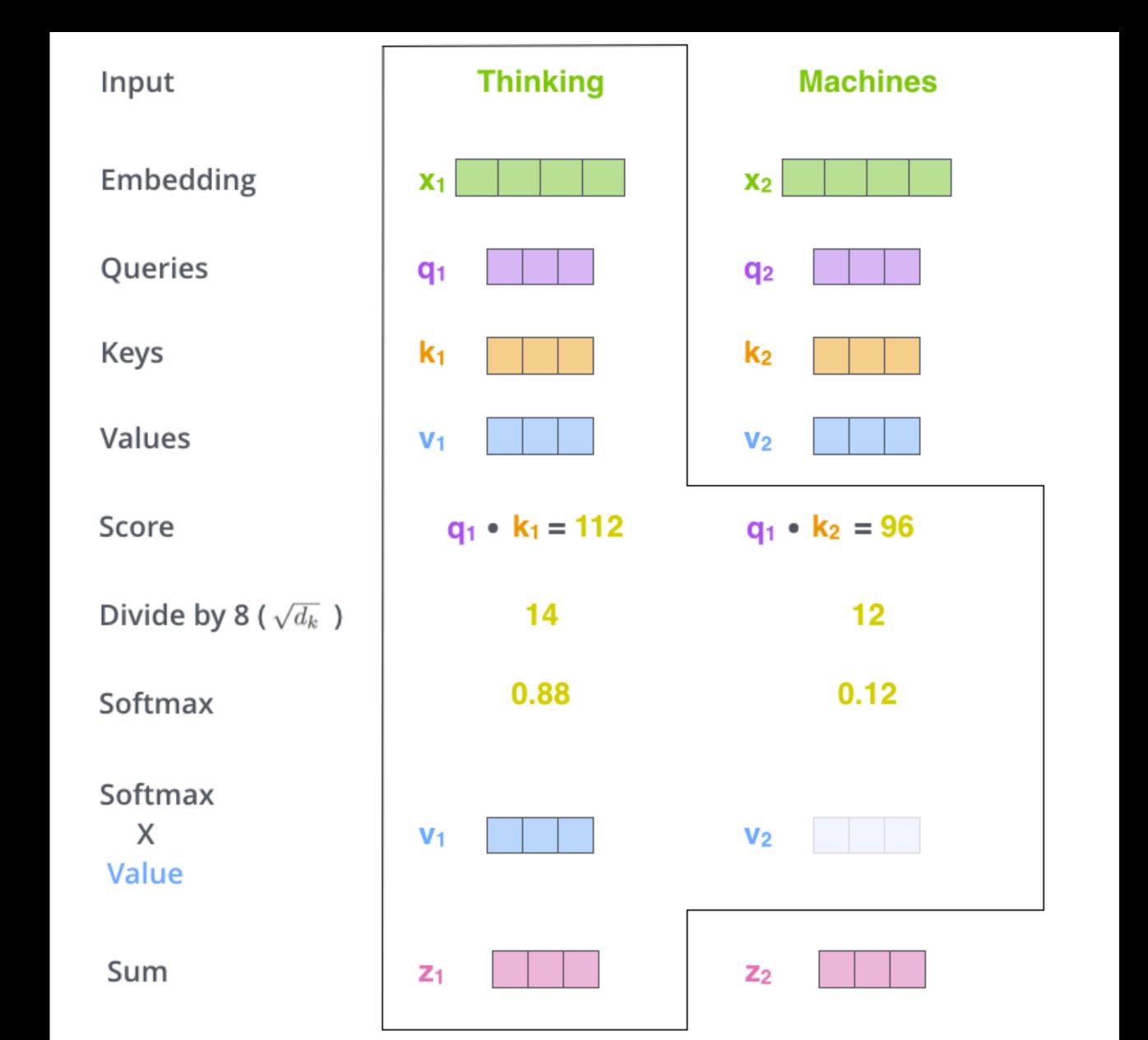
k₂

V₂

 $q_1 \cdot k_2 = 96$

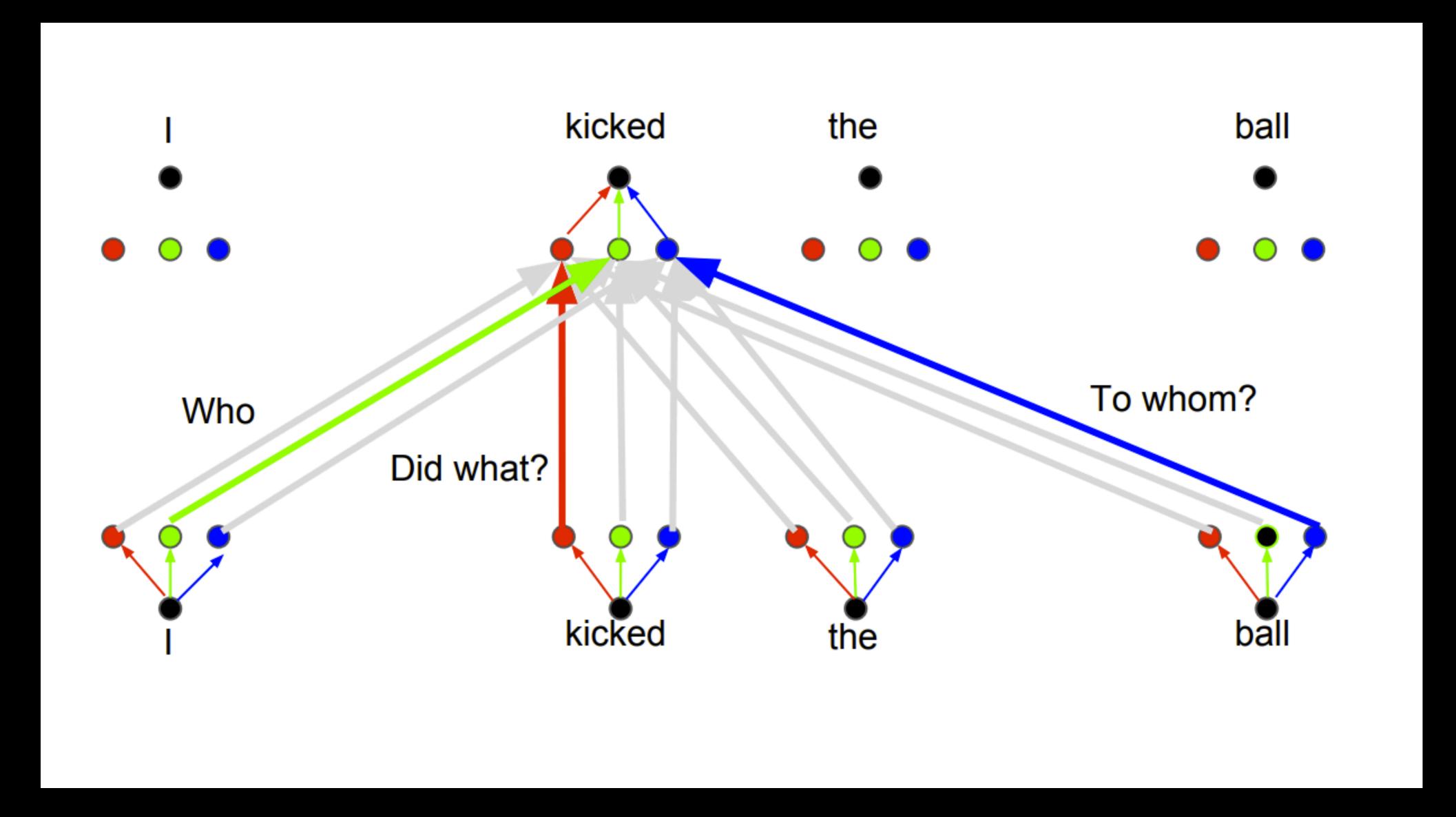
12

0.12



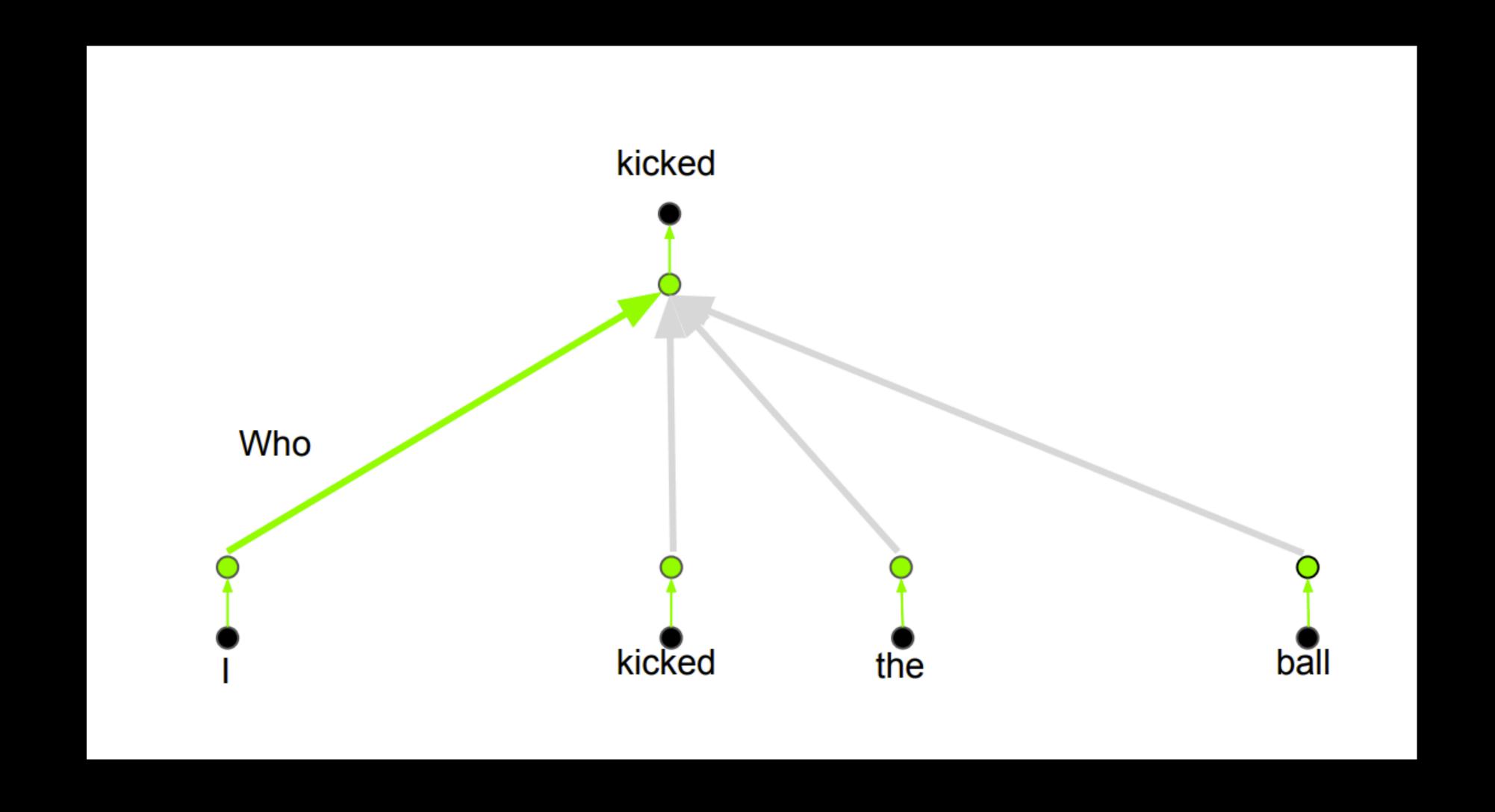


Self-attention multi-head



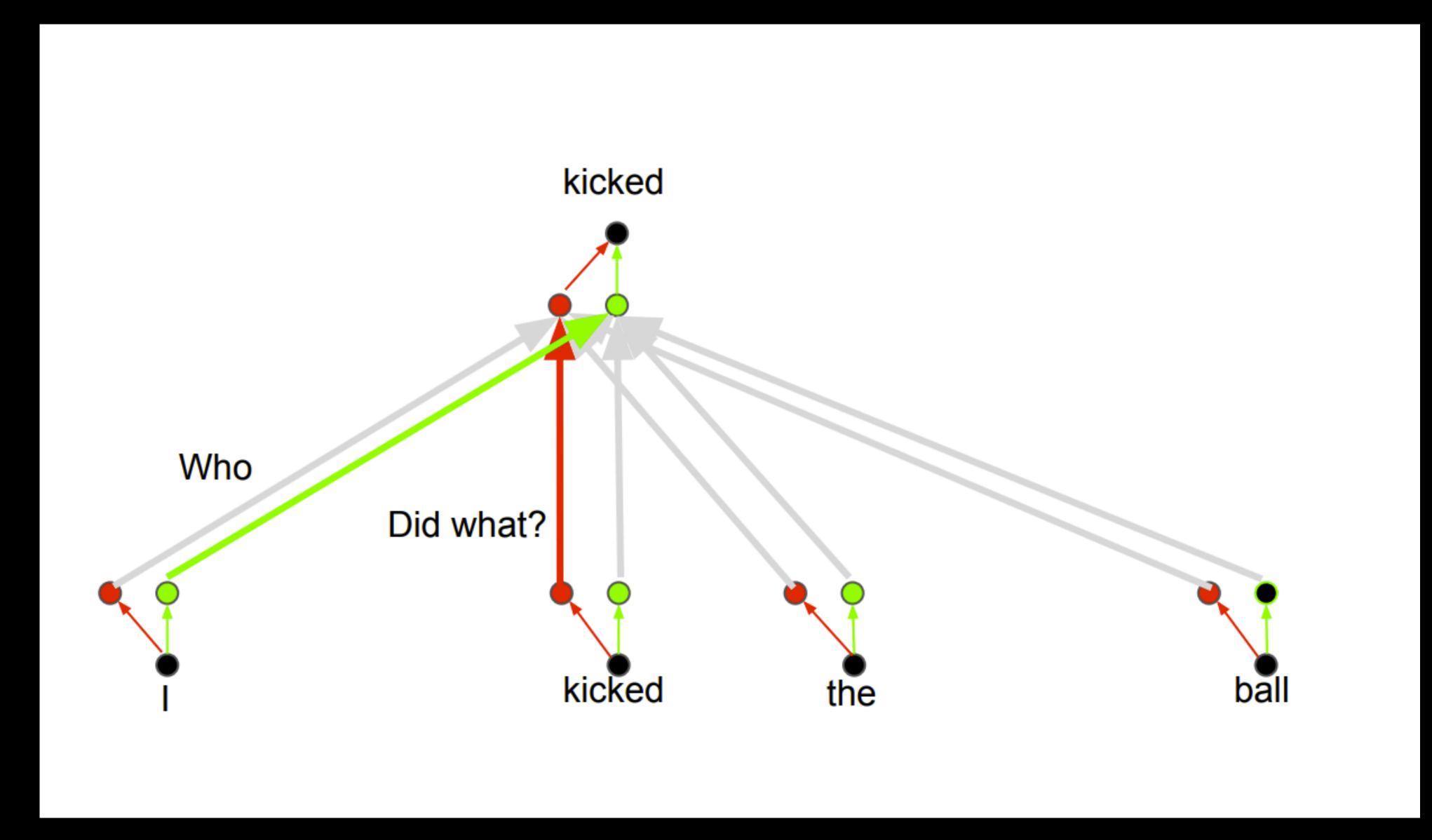


Self-attention multi-head - role of attention heads



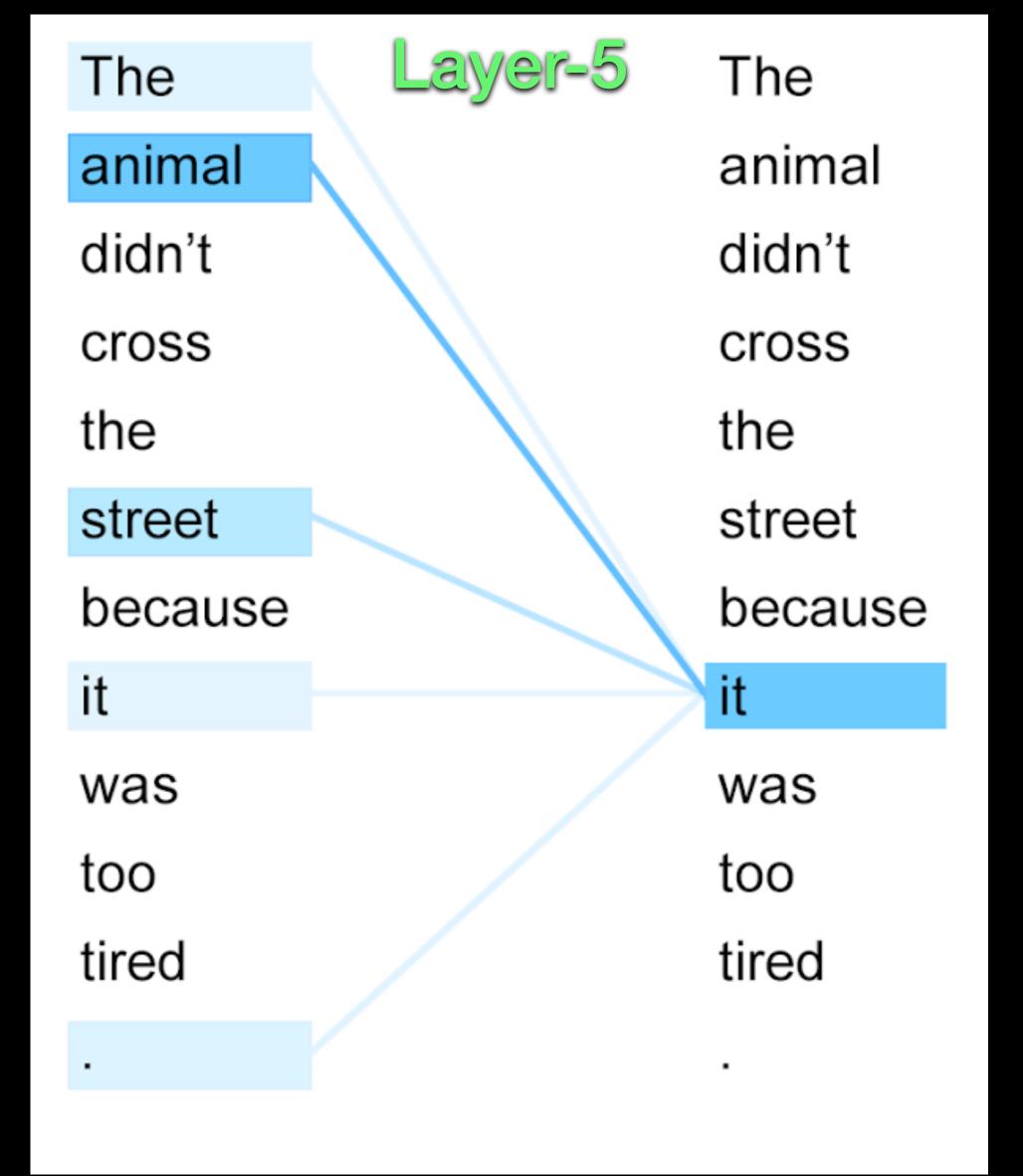


Self-attention multi-head - role of attention heads





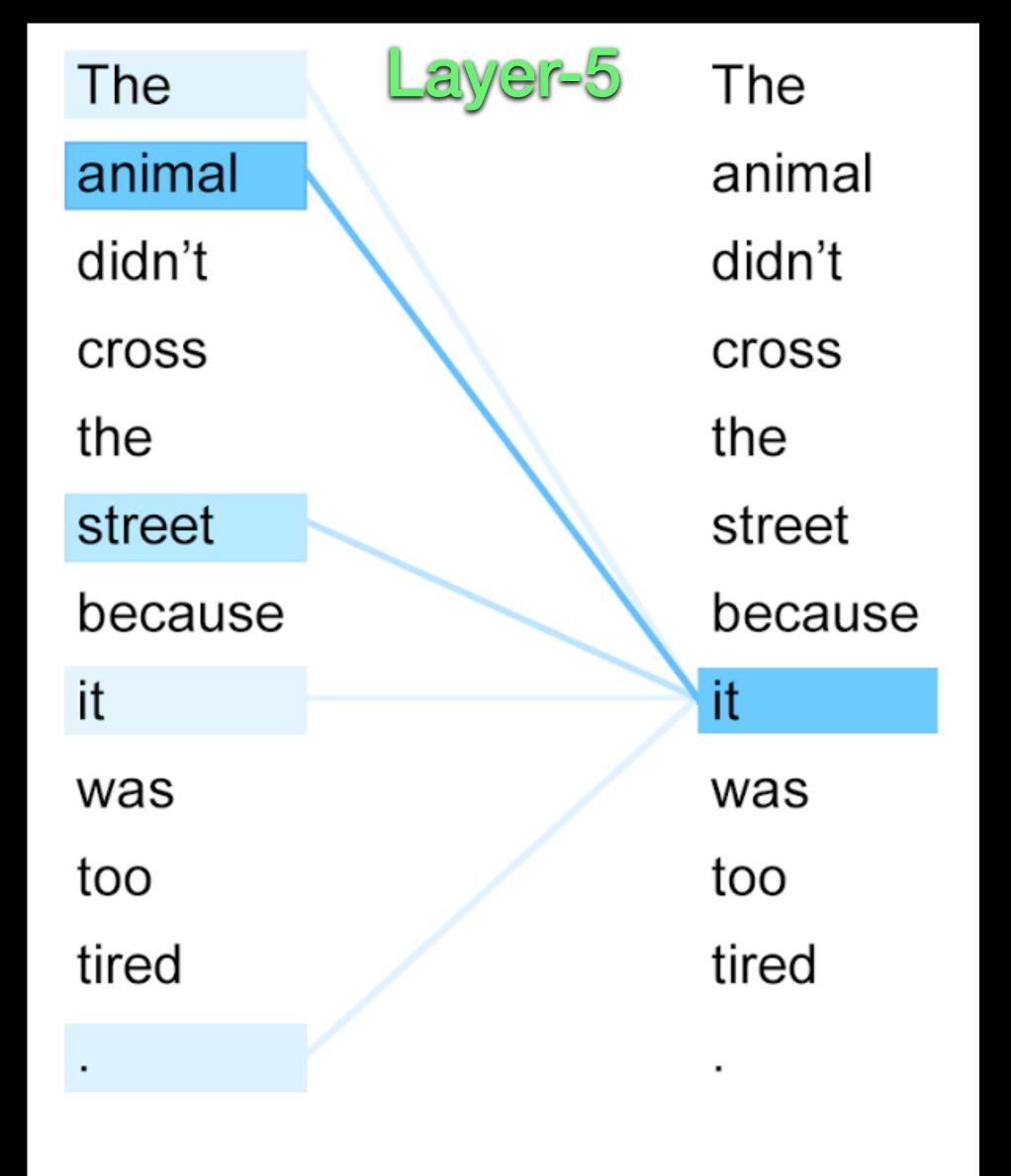
Self-attention - need for depth





Self-attention - need for depth

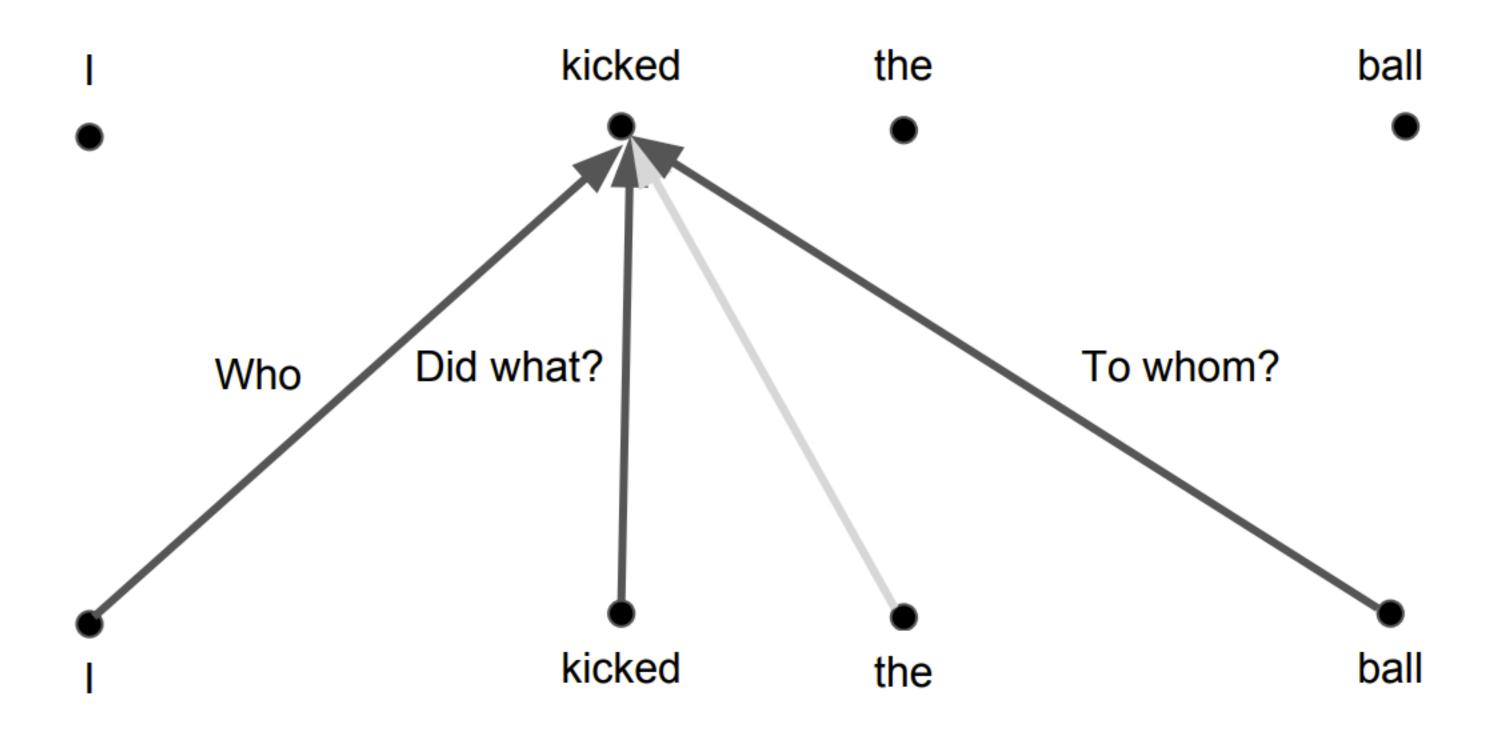
Layer-6					
The		The			
animal		animal			
didn't		didn't			
cross		cross			
the		the			
street		street			
because		because			
it		it			
was		was			
too		too			
wide		wide			





Need for multi-head attention

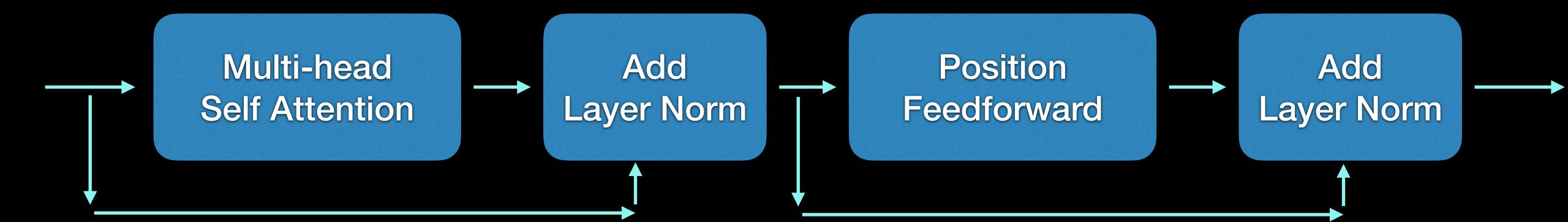






Single layer of encoder (typical implementation)

* Single encoder layer has typically self-attention skip connection, layer norm and feedforward layer





Positional encoding

* No recurrence or position awareness yet in the model

Binary format position can encode the rate of
change of bits across time

In floating format - one can use sines and cosines

```
0 0 0
                 1 0 0 0
                 1 0 0 1
0 0 0 1
           10:
                 1 0 1 0
           11:
                 1 0 1 1
           12:
           13:
           15:
```



Positional encoding

* An example used in the first paper

$$\mathbf{p}(t) \in \mathcal{R}^{D}$$

$$p_{i}(t) = \begin{cases} \sin(\omega_{k}t), & \text{if } i = 2k \\ \cos(\omega_{k}t), & \text{if } i = 2k + 1 \end{cases} \quad \mathbf{k} \in \{1...\frac{D}{2}\}$$

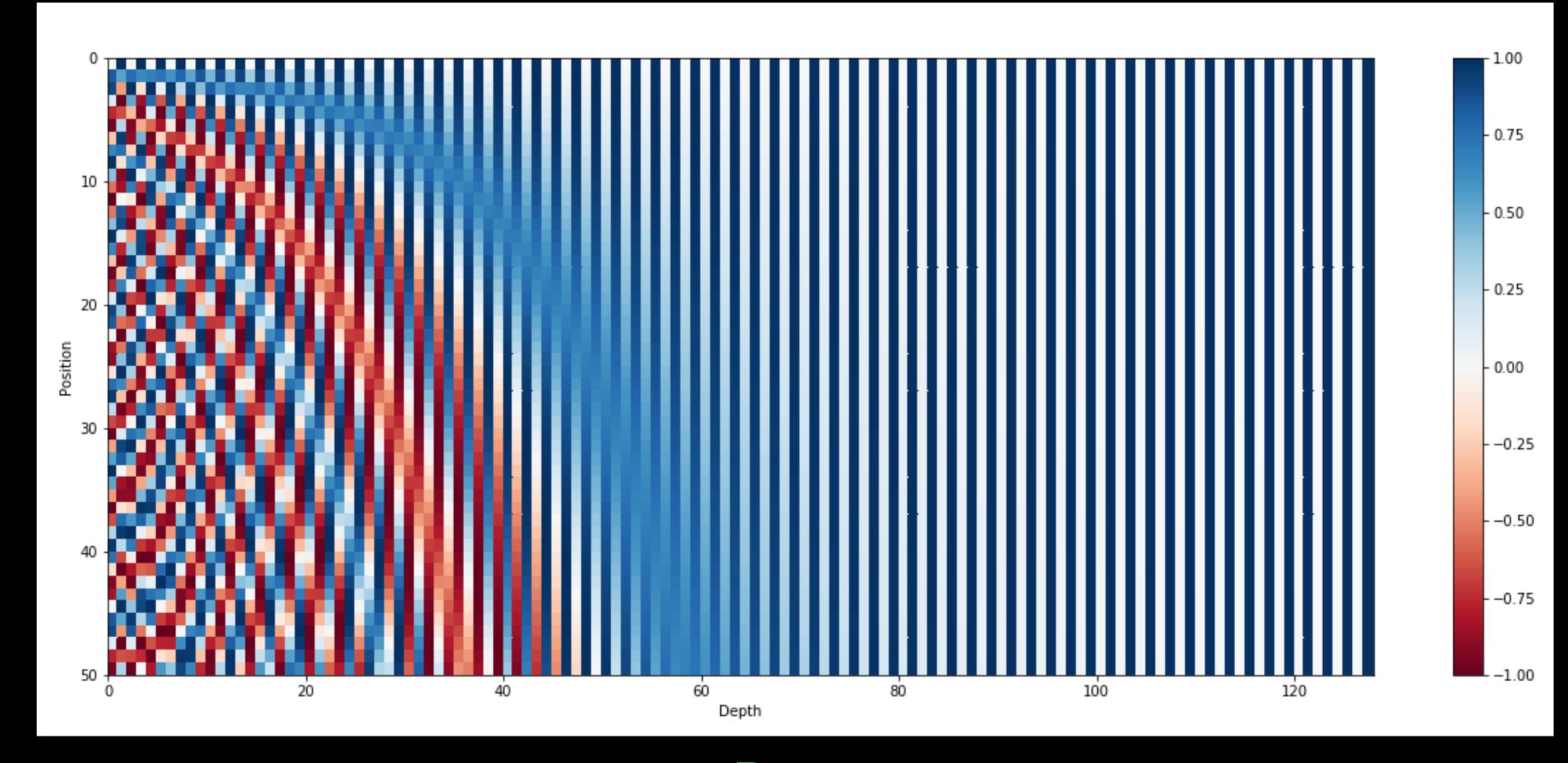
$$\omega_{k} = \frac{1}{10000^{\frac{2k}{D}}}$$

$$\mathbf{x}(t) = \mathbf{x}(t) + \mathbf{p}(t)$$



Positional encoding

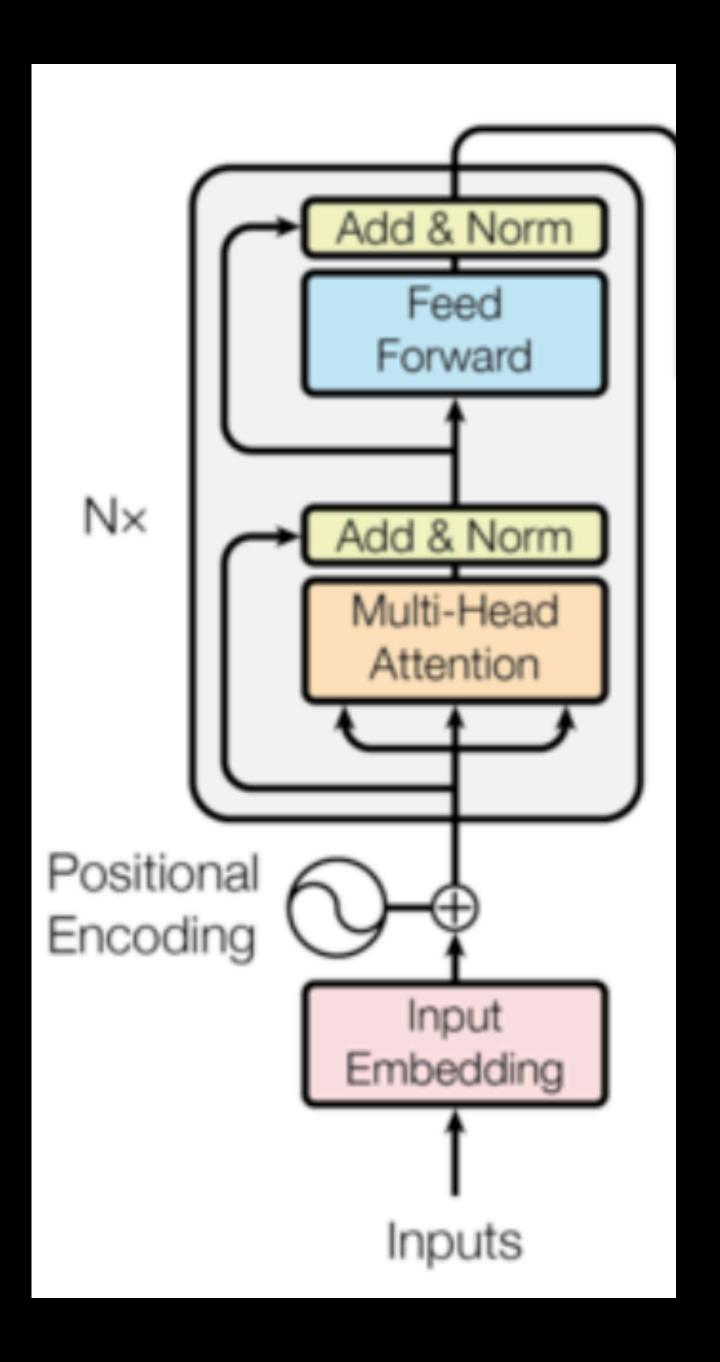
* An example used in the first paper $\mathbf{p}(t) \in \mathcal{R}^D$ [T=50, D=128]





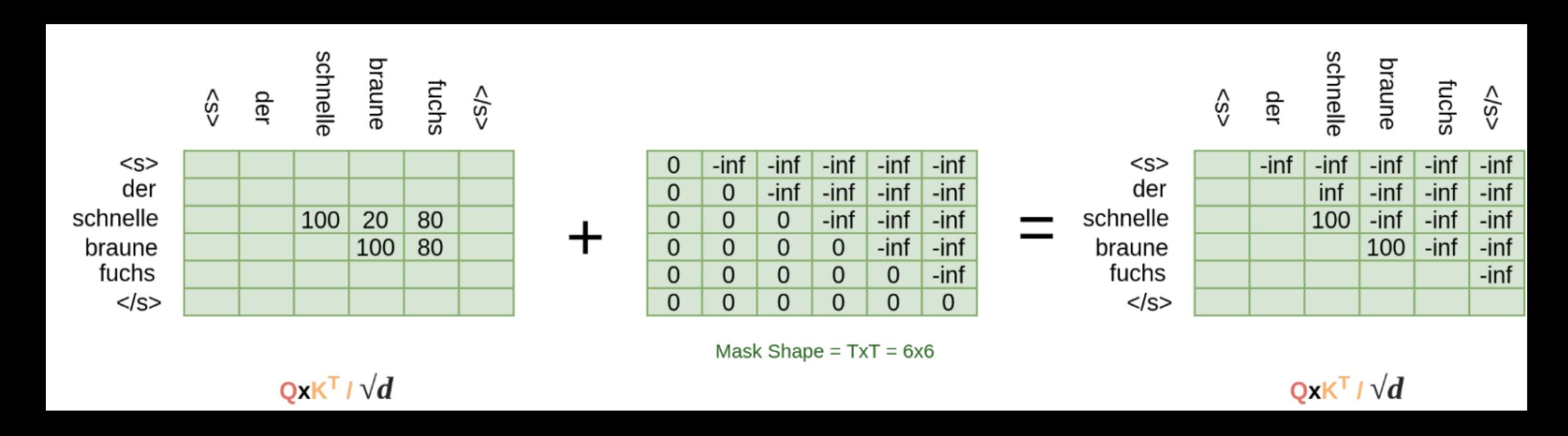
Transformer encoder - overview

Reading Assignment - "Attention is All You Need" https://arxiv.org/pdf/1706.03762.pdf





- * Masked self-attention layer -
 - Mask makes the output dependecies causal
 - * Only the past is used to encode the attention.





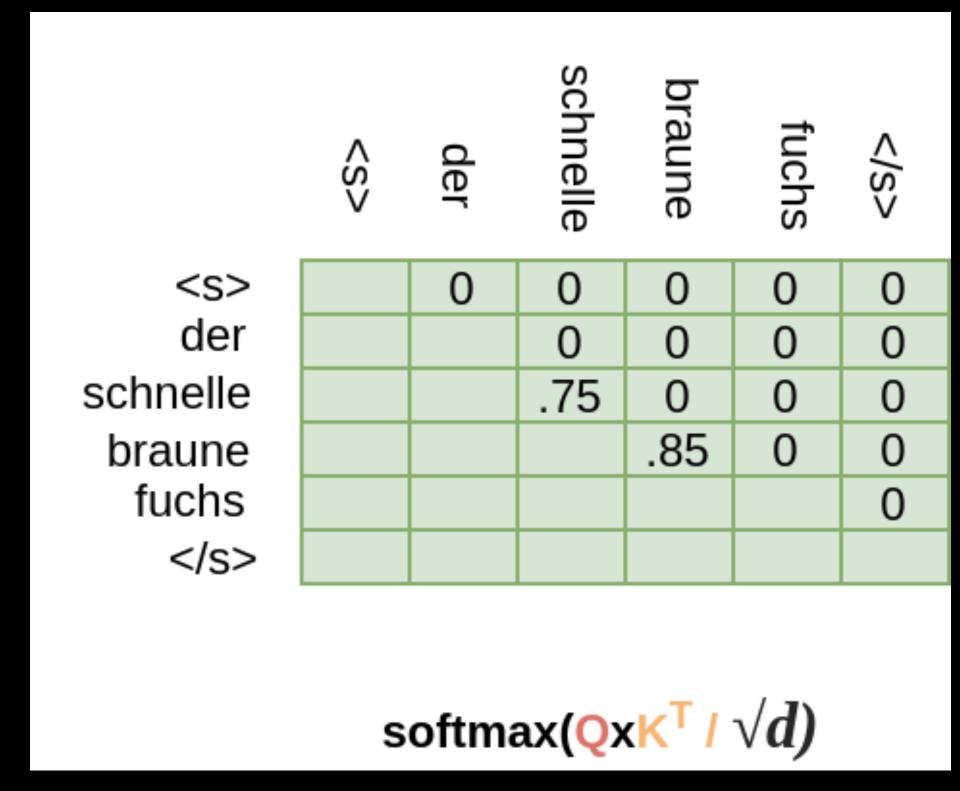
- * Masked self-attention layer -
 - ✓ Mask makes the output dependecies causal
 - * Only the past is used to encode the attention.

$$Softmax \left\{ \frac{\mathbf{Q}\mathbf{K}^T}{\sqrt{d}} \right\} \mathbf{V} \longrightarrow Softmax \left\{ Mask + \frac{\mathbf{Q}\mathbf{K}^T}{\sqrt{d}} \right\} \mathbf{V}$$

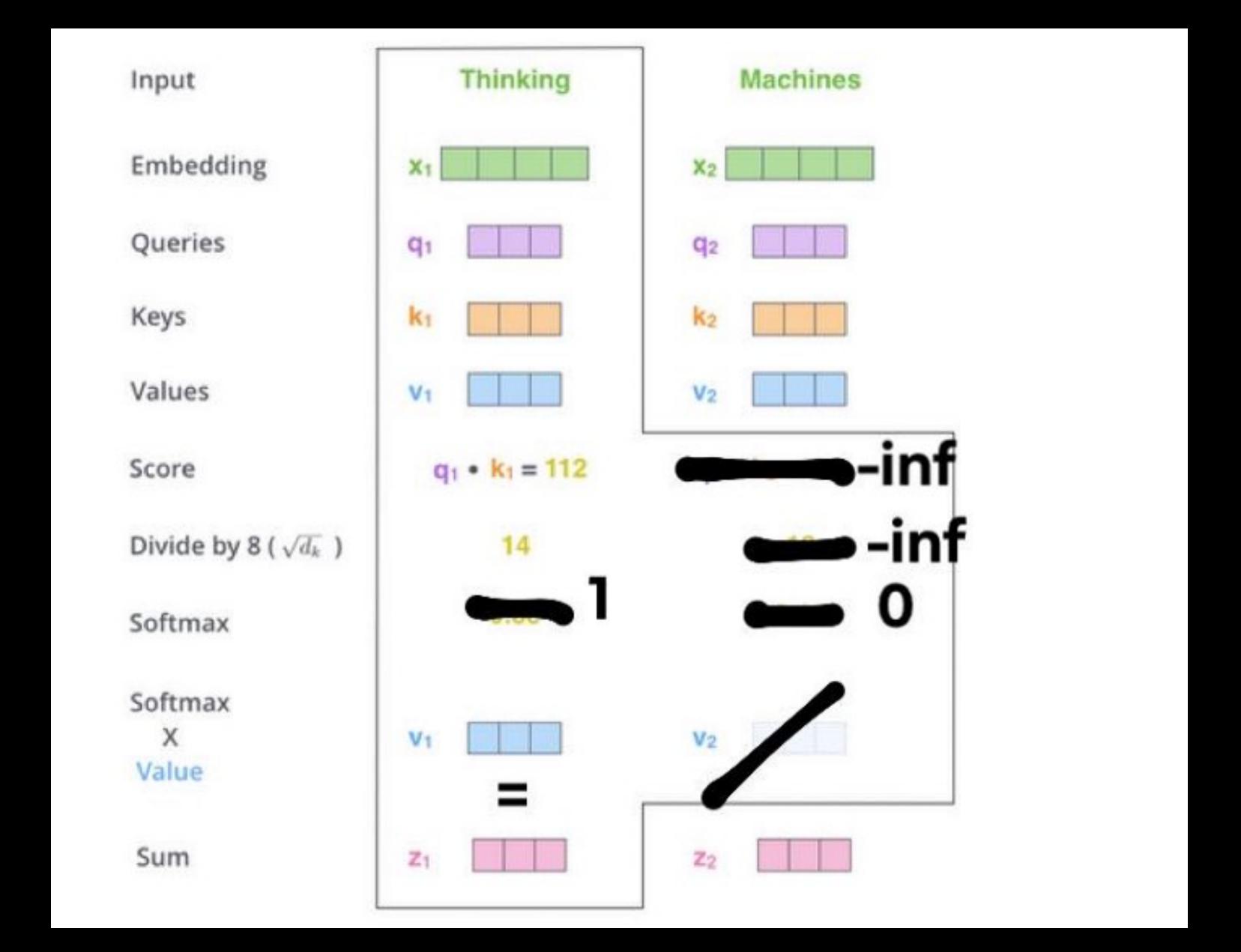
* Make the attention matrix to be lower triangular



- * Masked self-attention layer -
 - ✓ Mask makes the output dependecies causal
 - * Only the past is used to encode the attention.









Encoder-decoder attention

* Use the Key and Value matrices from the last layer of the encoder

$$\mathbf{Q}_{h}^{p} = \overline{\mathbf{D}}^{p-1} \mathbf{W}_{h}^{p,Q} + \mathbf{1} (\mathbf{b}_{h}^{p,Q})^{T} \in \mathcal{R}^{S \times d}$$

$$\mathbf{K}_{h}^{p} = \mathbf{E}^{L} \mathbf{W}_{h}^{p,K} + \mathbf{1} (\mathbf{b}_{h}^{p,K})^{T} \in \mathcal{R}^{T \times d}$$

$$\mathbf{V}_{h}^{p} = \mathbf{E}^{L} \mathbf{W}_{h}^{p,V} + \mathbf{1} (\mathbf{b}_{h}^{p,V})^{T} \in \mathcal{R}^{T \times d}$$

$$\mathbf{D}_{h}^{p} = softmax \left(\frac{\mathbf{Q}_{h}^{p} (\mathbf{K}_{h}^{p})^{T}}{\sqrt{d}}\right) \mathbf{V}_{h}^{p} \in \mathcal{R}^{S \times d}$$

$$h=\{1..H\}$$
 heads $d=rac{D}{H}$



* Decoder Layer Output

*
$$\left[\mathbf{d}^{p}(1)...\mathbf{d}^{p}(S)\right] = ReLU\left(\mathbf{D}_{ff}^{p}\mathbf{W}_{of}^{p} + \mathbf{1}(\mathbf{b}_{of}^{p})^{T}\right) \in \mathcal{R}^{S \times D}$$



Transformer - full pipeline

