E9 205 Machine Learning for Signal Processing

Linear Models for Regression and Classification

23-09-2019

Established 1911

INDIAN INSTITUTE OF SCIENCE
भारतीय विज्ञान संस्थान

Linear Regression

$$y(\mathbf{x}, \mathbf{w}) = \sum_{j=0}^{M-1} w_j \phi_j(\mathbf{x}) = \mathbf{w}^{\mathrm{T}} \boldsymbol{\phi}(\mathbf{x})$$

$$t = y(\mathbf{x}, \mathbf{w}) + \epsilon$$

- Solution to Maximum Likelihood problem is the least squares solution

$$\nabla \ln p(\mathbf{t}|\mathbf{w}, \beta) = \sum_{n=1}^{N} \left\{ t_n - \mathbf{w}^{\mathrm{T}} \boldsymbol{\phi}(\mathbf{x}_n) \right\} \boldsymbol{\phi}(\mathbf{x}_n)^{\mathrm{T}}.$$

Pseudo Inverse Based Solution

Bishop - PRML book (Chap 3)

Established 1911

INDIAN INSTITUTE OF SCIENCE
भारतीय विज्ञान संस्थान

Choice of Basis Functions

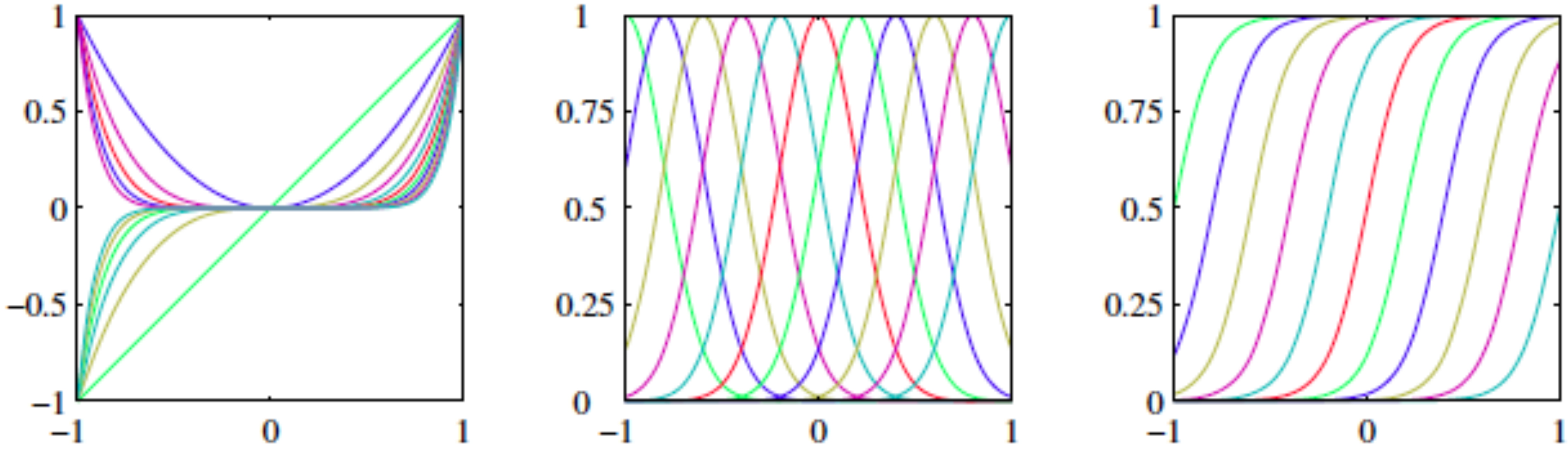
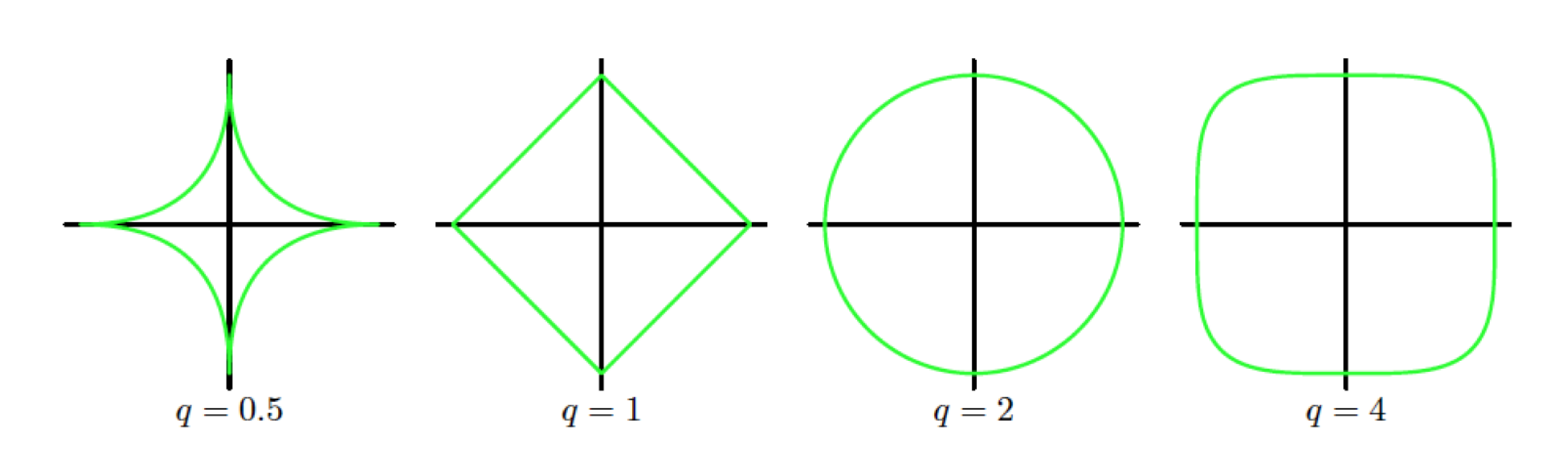


Figure 3.1 Examples of basis functions, showing polynomials on the left, Gaussians of the form (3.4) in the centre, and sigmoidal of the form (3.5) on the right.

Regularized Least Squares

- Optimize a modified cost function $E_D(\mathbf{w}) + \lambda E_W(\mathbf{w})$

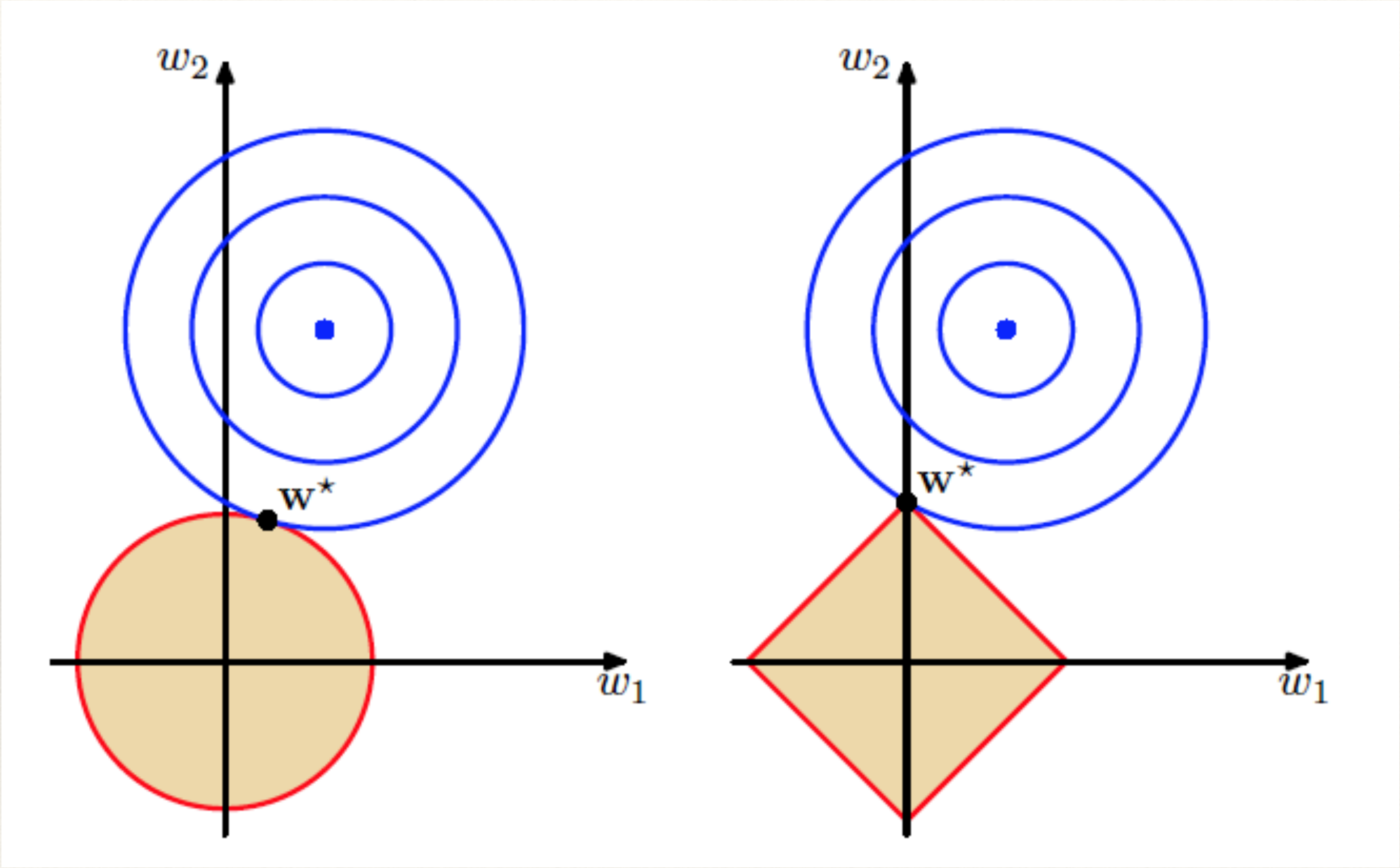


Bishop - PRML book (Chap 3)

Established 1911

INDIAN INSTITUTE OF SCIENCE
भारतीय विज्ञान संस्थान

Regularized Least Squares



Bishop - PRML book (Chap 3)

Choice of Regularization Parameter

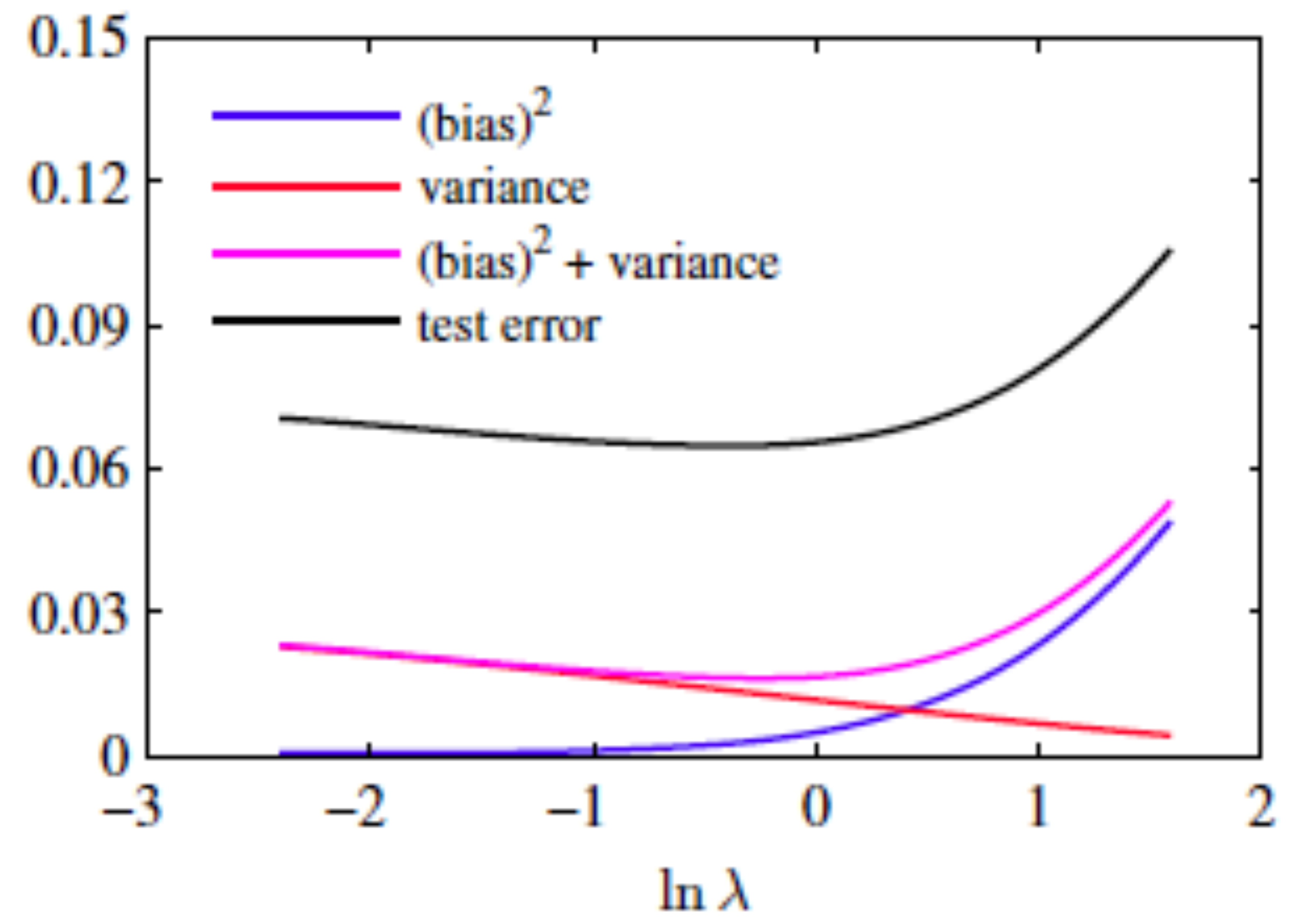


Figure 3.5 Illustration of the dependence of bias and variance on model complexity, governed by a regularization parameter λ, using the sinusoidal data set from Chapter 1. There are $L = 100$ data sets, each having $N = 25$ data points, and there are 24 Gaussian basis functions in the model so that the total number of parameters is $M = 25$ including the bias parameter. The left column shows the result of fitting the model to the data sets for various values of $\ln \lambda$ (for clarity, only 20 of the 100 fits are shown). The right column shows the corresponding average of the 100 fits (red) along with the sinusoidal function from which the data sets were generated (green).

Choice of Regularization Parameter