E9 205 Machine Learning for Signal Processing

Probablistic Linear Models

30-09-2019

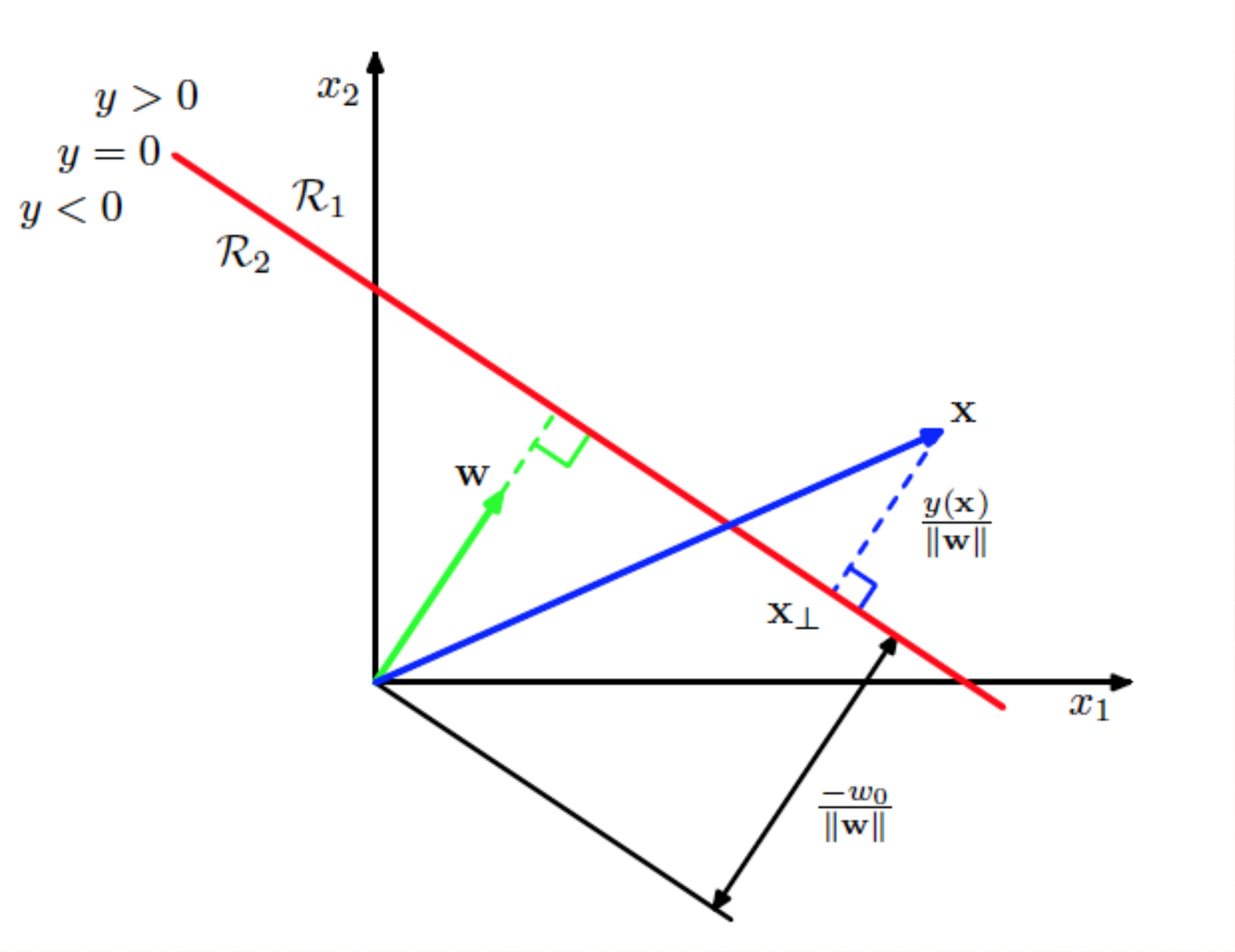
Established
1911

INDIAN INSTITUTE OF SCIENCE
भारतीय विज्ञान संस्थान

Linear Models for Classification

- Optimize a modified cost function

$$y(\mathbf{x}) = \mathbf{w}^{\mathrm{T}}\mathbf{x} + w_0$$



Established 1911

INDIAN INSTITUTE OF SCIENCE
भारतीय विज्ञान संस्थान

Least Squares for Classification

- K-class classification problem

$$y_k(\mathbf{x}) = \mathbf{w}_k^{\mathrm{T}}\mathbf{x} + w_{k0}$$

$$\mathbf{y}(\mathbf{x}) = \widetilde{\mathbf{W}}^{\mathrm{T}}\widetilde{\mathbf{x}}$$

- With 1-of-K hot encoding, and least squares regression

$$E_D(\widetilde{\mathbf{W}}) = \frac{1}{2}\mathrm{Tr}\left\{(\widetilde{\mathbf{X}}\widetilde{\mathbf{W}} - \mathbf{T})^{\mathrm{T}}(\widetilde{\mathbf{X}}\widetilde{\mathbf{W}} - \mathbf{T})\right\}$$

Logistic Regression

- 2- class logistic regression

$$p(\mathcal{C}_1|\phi) = y(\phi) = \sigma\left(\mathbf{w}^{\mathrm{T}}\phi\right)$$

- Maximum likelihood solution

$$\nabla E(\mathbf{w}) = \sum_{n=1}^{N}(y_n - t_n)\phi_n$$

- K-class logistic regression

$$p(\mathcal{C}_k|\phi) = y_k(\phi) = \frac{\exp(a_k)}{\sum_j \exp(a_j)}$$

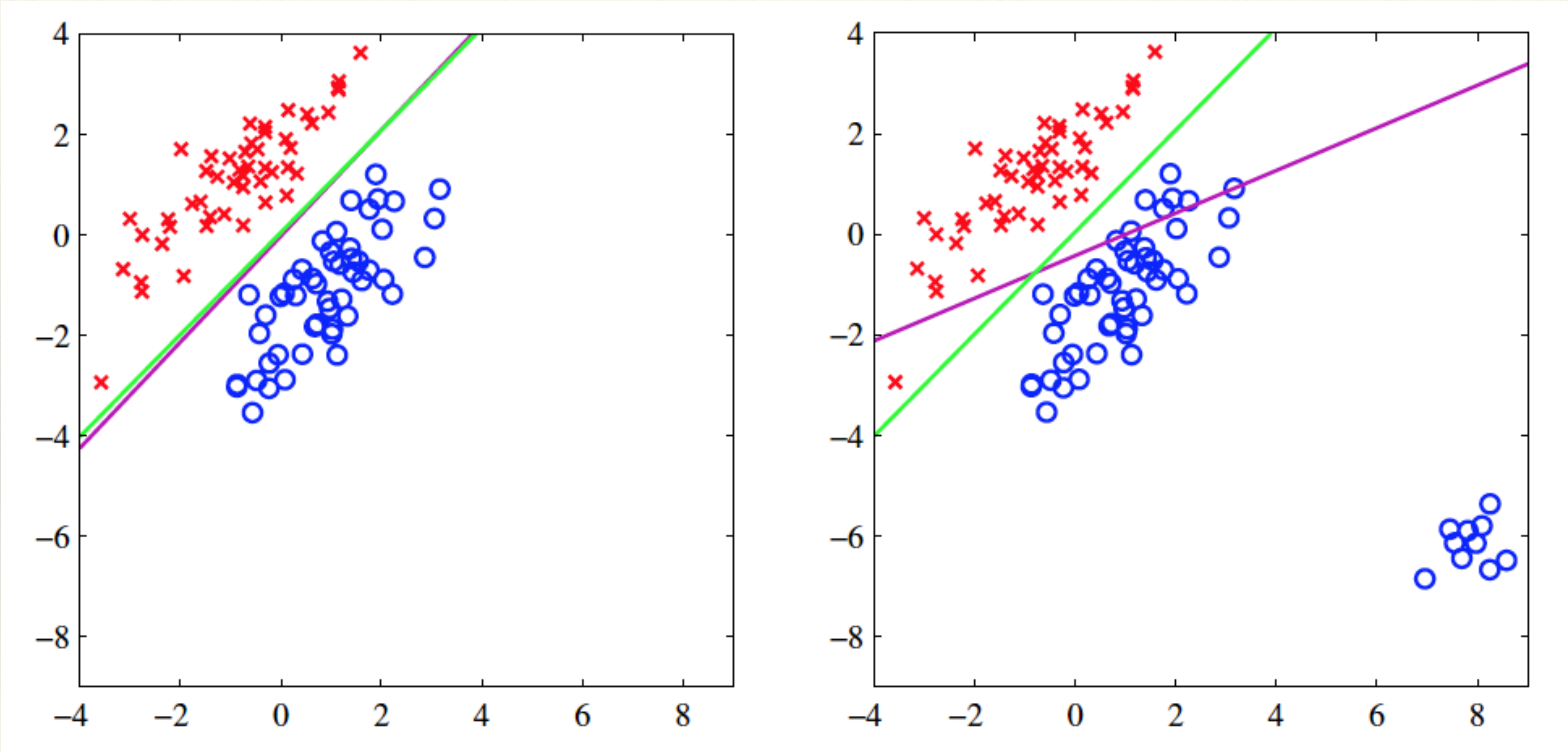
$$a_k = \mathbf{w}_k^{\mathrm{T}}\phi.$$

- Maximum likelihood solution

$$\nabla_{\mathbf{w}_j} E(\mathbf{w}_1, \ldots, \mathbf{w}_K) = \sum_{n=1}^{N}(y_{nj} - t_{nj})\,\phi_n$$

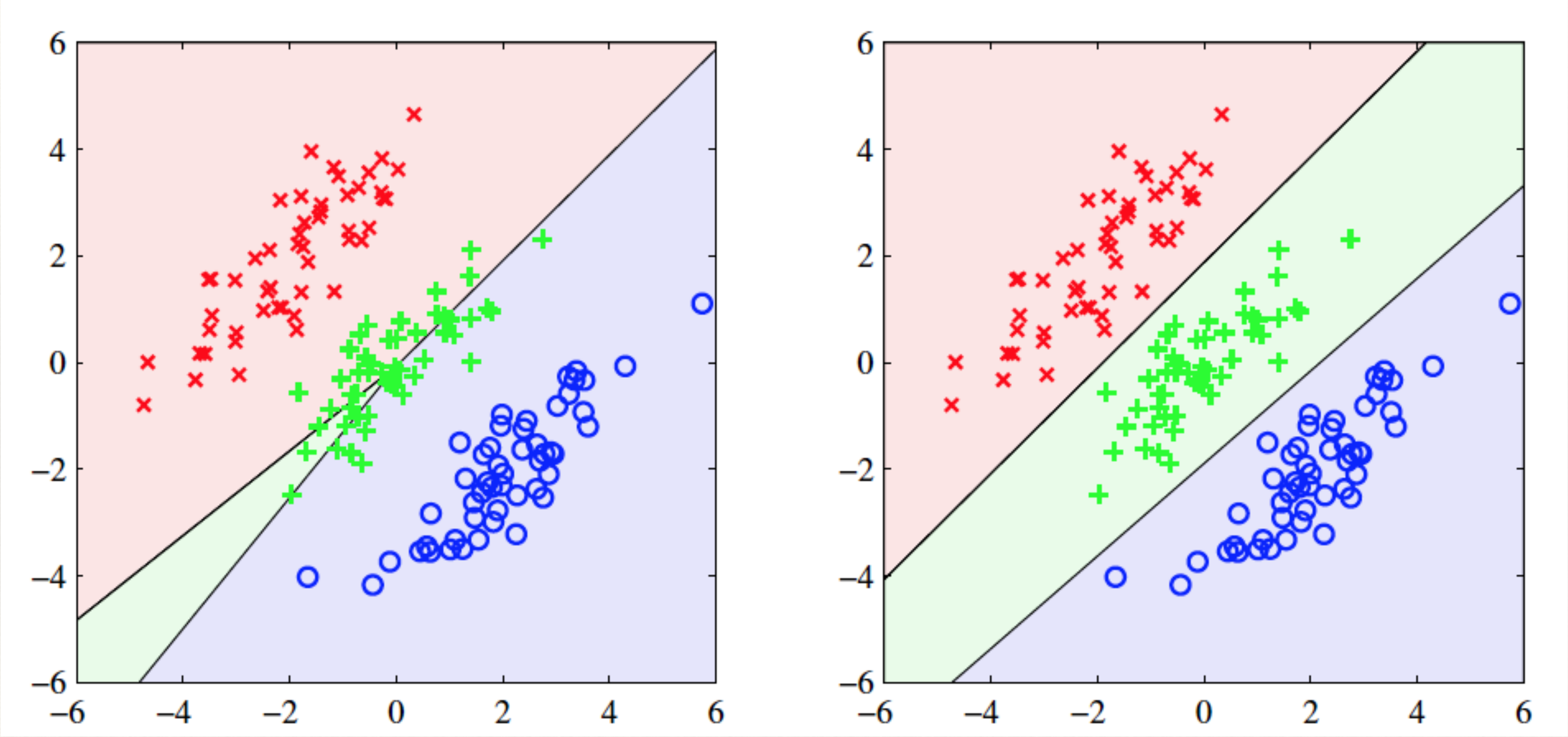
Bishop - PRML book (Chap 3)

Least Squares versus Logistic Regression



Bishop - PRML book (Chap 4)

Least Squares versus Logistic Regression



Bishop - PRML book (Chap 4)