Machine Learning for Signal Processing

Recurrent Neural Networks

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Introduction

- The standard DNN/CNN paradigms
  - (x,y) - ordered pair of data vectors/images (x) and target (y)
- Moving to sequence data
  - (x(t),y(t)) where this could be sequence to sequence mapping task.
  - (x(t),y) where this could be a sequence to vector mapping task.

Introduction

- Difference between CNNs/DNNs
  - (x(t),y(t)) where this could be sequence to sequence mapping task.
    - Input features / output targets are correlated in time.
    - Unlike standard models where each pair is independent.
    - Need to model dependencies in the sequence over time.

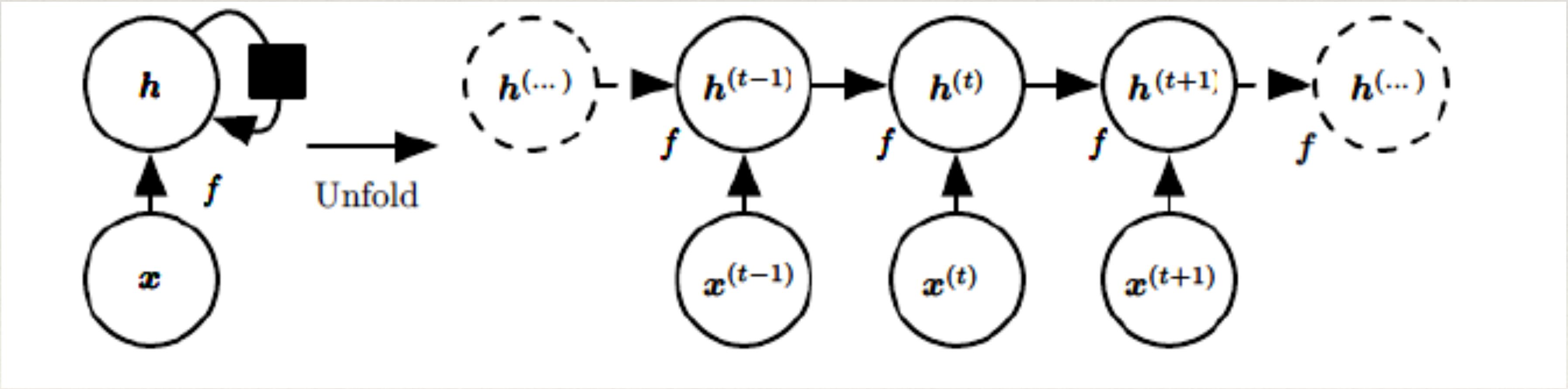
Introduction to Recurrent Networks

$$s^{(t)} = f(s^{(t-1)}; \theta),$$

$$s^{(3)} = f(s^{(2)}; \theta) = f(f(s^{(1)}; \theta); \theta)$$

$$s^{(t)} = f(s^{(t-1)}, x^{(t)}; \theta),$$

$$h^{(t)} = f(h^{(t-1)}, x^{(t)}; \theta),$$



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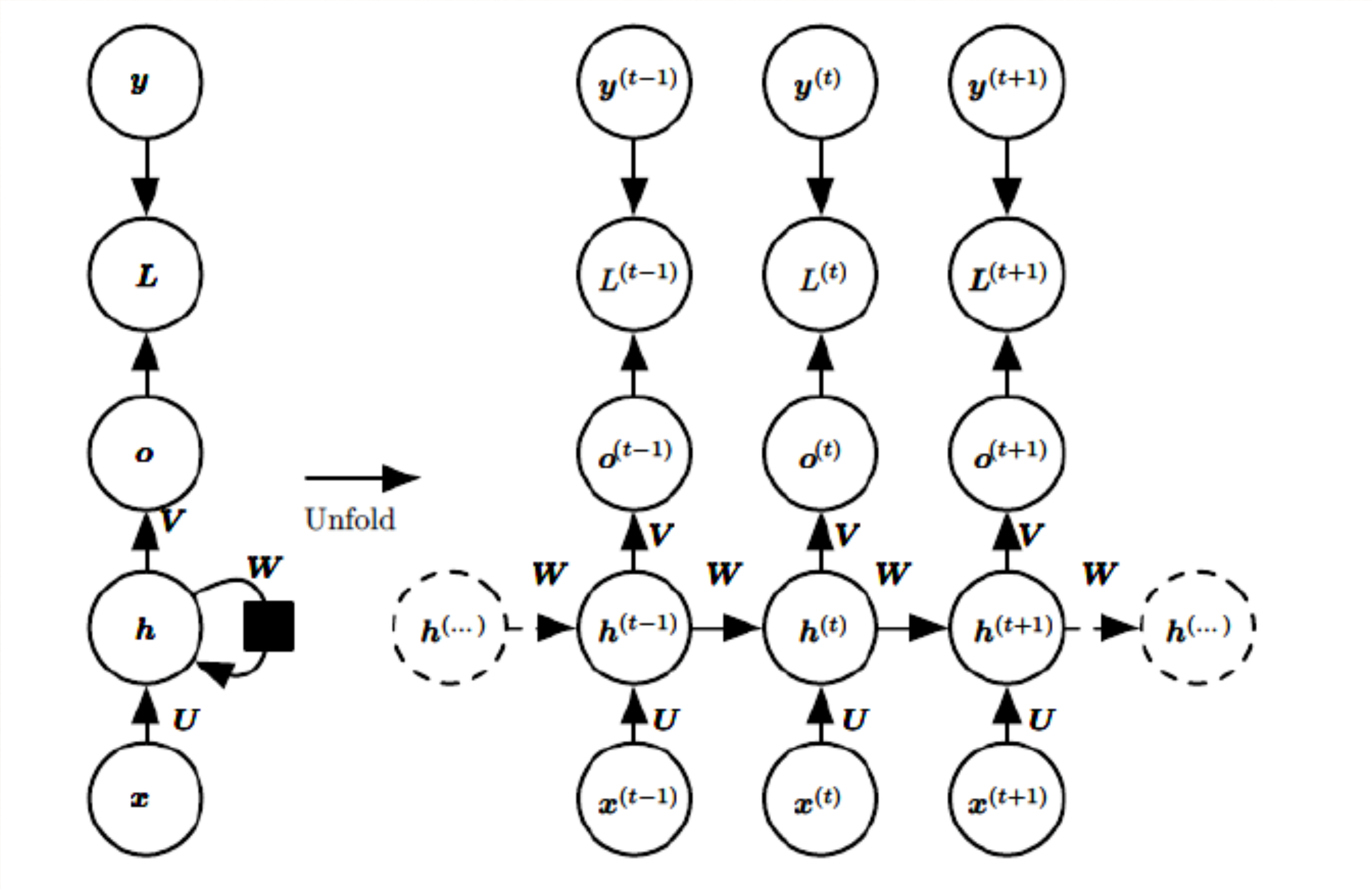
Recurrent Networks



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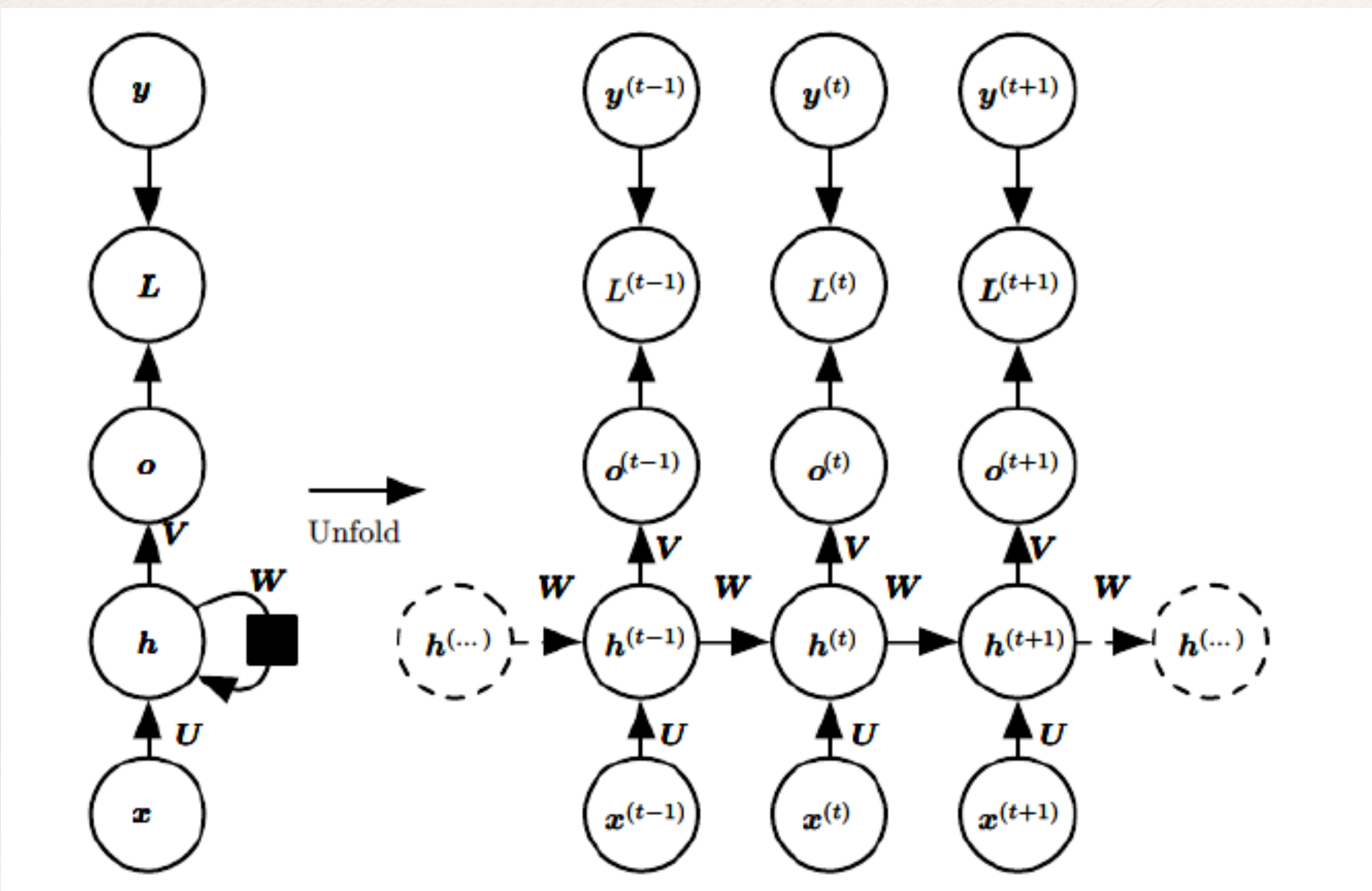
Recurrent Networks



$$
\begin{aligned}
\boldsymbol{a}^{(t)} &= \boldsymbol{b} + \boldsymbol{W}\boldsymbol{h}^{(t-1)} + \boldsymbol{U}\boldsymbol{x}^{(t)} \\
\boldsymbol{h}^{(t)} &= \tanh(\boldsymbol{a}^{(t)}) \\
\boldsymbol{o}^{(t)} &= \boldsymbol{c} + \boldsymbol{V}\boldsymbol{h}^{(t)} \\
\hat{\boldsymbol{y}}^{(t)} &= \mathrm{softmax}(\boldsymbol{o}^{(t)})
\end{aligned}
$$

$$
\begin{aligned}
& L\left(\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(\tau)}\}, \{\boldsymbol{y}^{(1)}, \ldots, \boldsymbol{y}^{(\tau)}\}\right) \\
&= \sum_t L^{(t)} \\
&= -\sum_t \log p_{\text{model}}\left(y^{(t)} \mid \{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(t)}\}\right)
\end{aligned}
$$

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Back Propagation in RNNs

$$
\begin{aligned}
\boldsymbol{a}^{(t)} &= \boldsymbol{b} + \boldsymbol{W}\boldsymbol{h}^{(t-1)} + \boldsymbol{U}\boldsymbol{x}^{(t)} \\
\boldsymbol{h}^{(t)} &= \tanh(\boldsymbol{a}^{(t)}) \\
\boldsymbol{o}^{(t)} &= \boldsymbol{c} + \boldsymbol{V}\boldsymbol{h}^{(t)} \\
\hat{\boldsymbol{y}}^{(t)} &= \operatorname{softmax}(\boldsymbol{o}^{(t)})
\end{aligned}
$$

Model Parameters

$\boldsymbol{U}$, $\boldsymbol{V}$, $\boldsymbol{W}$, $\boldsymbol{b}$ and $\boldsymbol{c}$

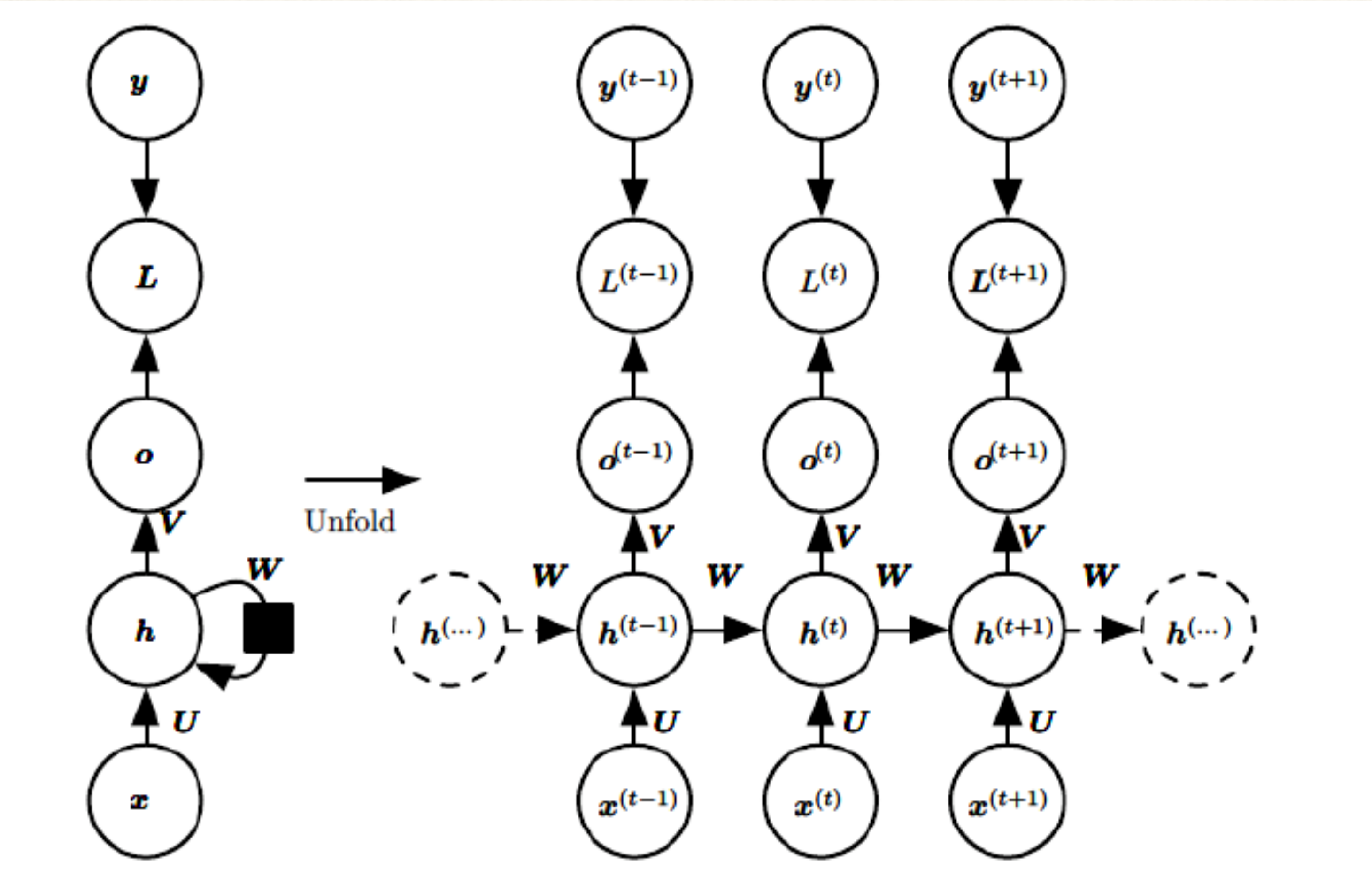
Gradient Descent

$$
\begin{aligned}
& L\left(\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(\tau)}\}, \{\boldsymbol{y}^{(1)}, \ldots, \boldsymbol{y}^{(\tau)}\}\right) \\
&= \sum_t L^{(t)} \\
&= -\sum_t \log p_{\text{model}}\left(y^{(t)} \mid \{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(t)}\}\right)
\end{aligned}
$$

$$
\frac{\partial L}{\partial L^{(t)}} = 1.
$$

$$
(\nabla_{\boldsymbol{o}^{(t)}} L)_i = \frac{\partial L}{\partial o_i^{(t)}} = \frac{\partial L}{\partial L^{(t)}} \frac{\partial L^{(t)}}{\partial o_i^{(t)}} = \hat{y}_i^{(t)} - \mathbf{1}_{i, y^{(t)}}
$$

Recurrent Networks



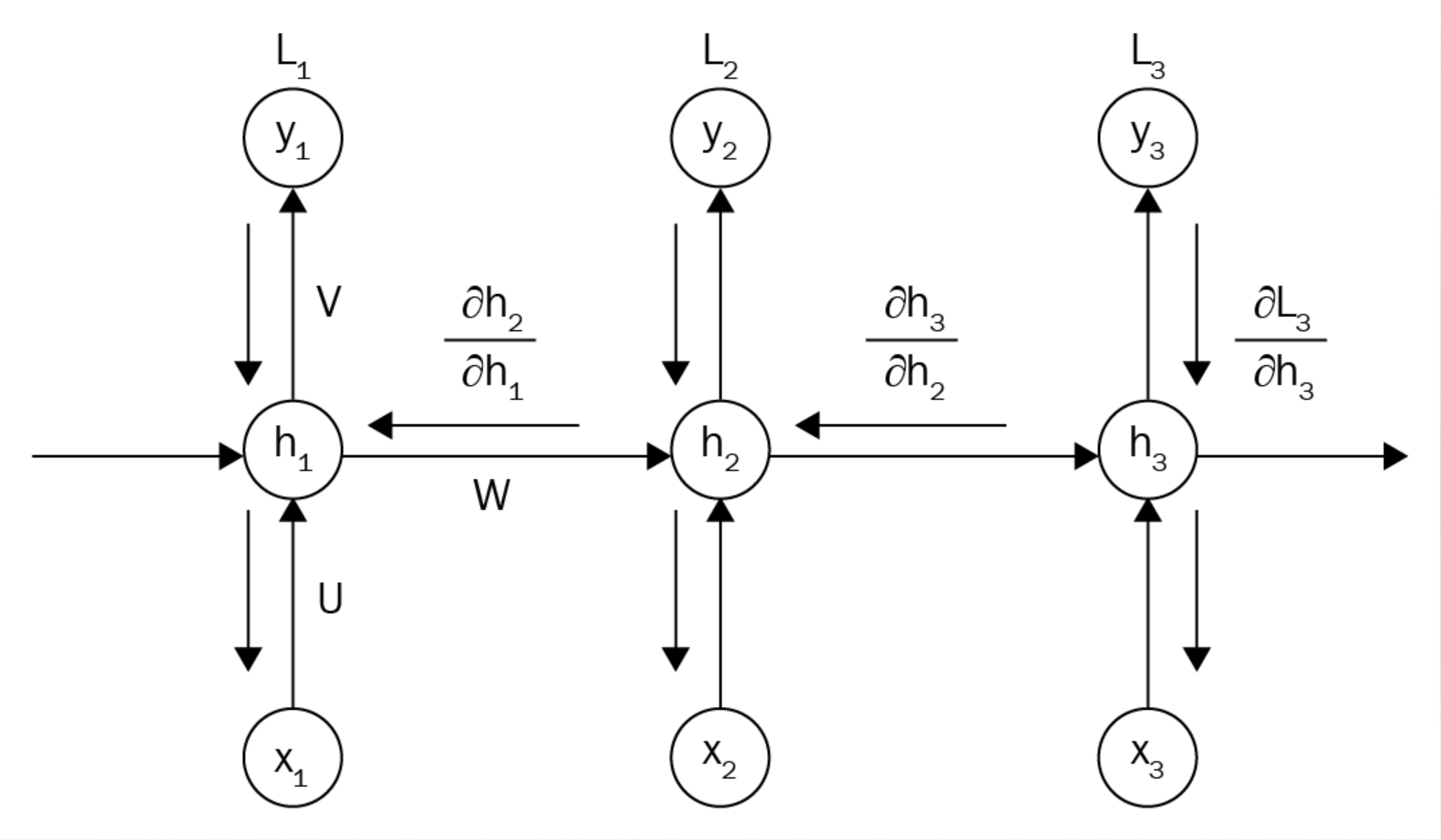
$$(\nabla_{o^{(t)}} L)_i = \frac{\partial L}{\partial o_i^{(t)}} = \frac{\partial L}{\partial L^{(t)}} \frac{\partial L^{(t)}}{\partial o_i^{(t)}} = \hat{y}_i^{(t)} - \mathbf{1}_{i,y^{(t)}}$$

$$\nabla_{\boldsymbol{h}^{(\tau)}} L = \boldsymbol{V}^\top \nabla_{\boldsymbol{o}^{(\tau)}} L.$$

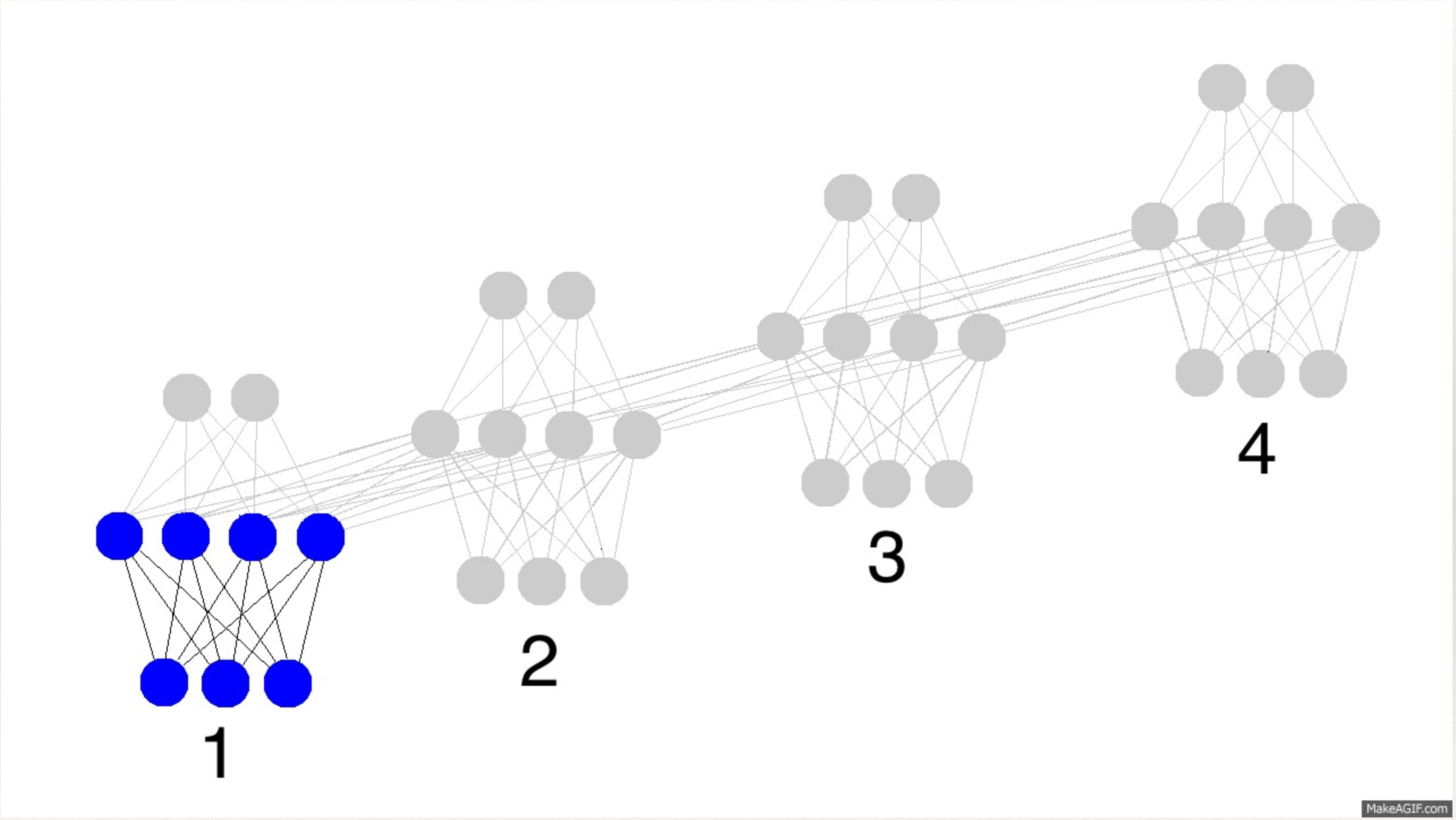
$$\nabla_{\boldsymbol{h}^{(t)}} L = \left(\frac{\partial \boldsymbol{h}^{(t+1)}}{\partial \boldsymbol{h}^{(t)}}\right)^\top (\nabla_{\boldsymbol{h}^{(t+1)}} L) + \left(\frac{\partial \boldsymbol{o}^{(t)}}{\partial \boldsymbol{h}^{(t)}}\right)^\top (\nabla_{\boldsymbol{o}^{(t)}} L)$$

$$= \boldsymbol{W}^\top (\nabla_{\boldsymbol{h}^{(t+1)}} L) \operatorname{diag}\left(1 - \left(\boldsymbol{h}^{(t+1)}\right)^2\right) + \boldsymbol{V}^\top (\nabla_{\boldsymbol{o}^{(t)}} L)$$

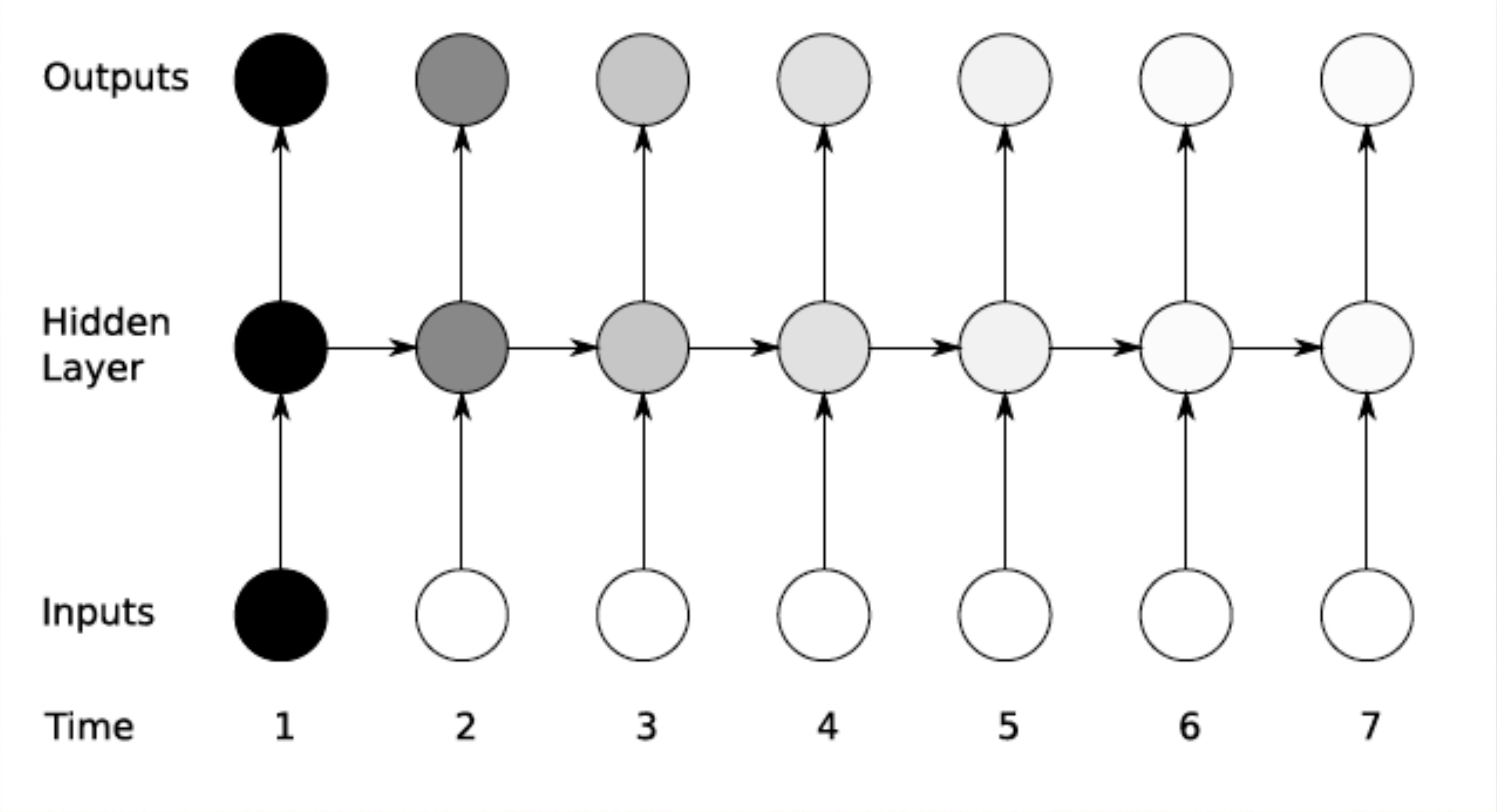
Back Propagation Through Time



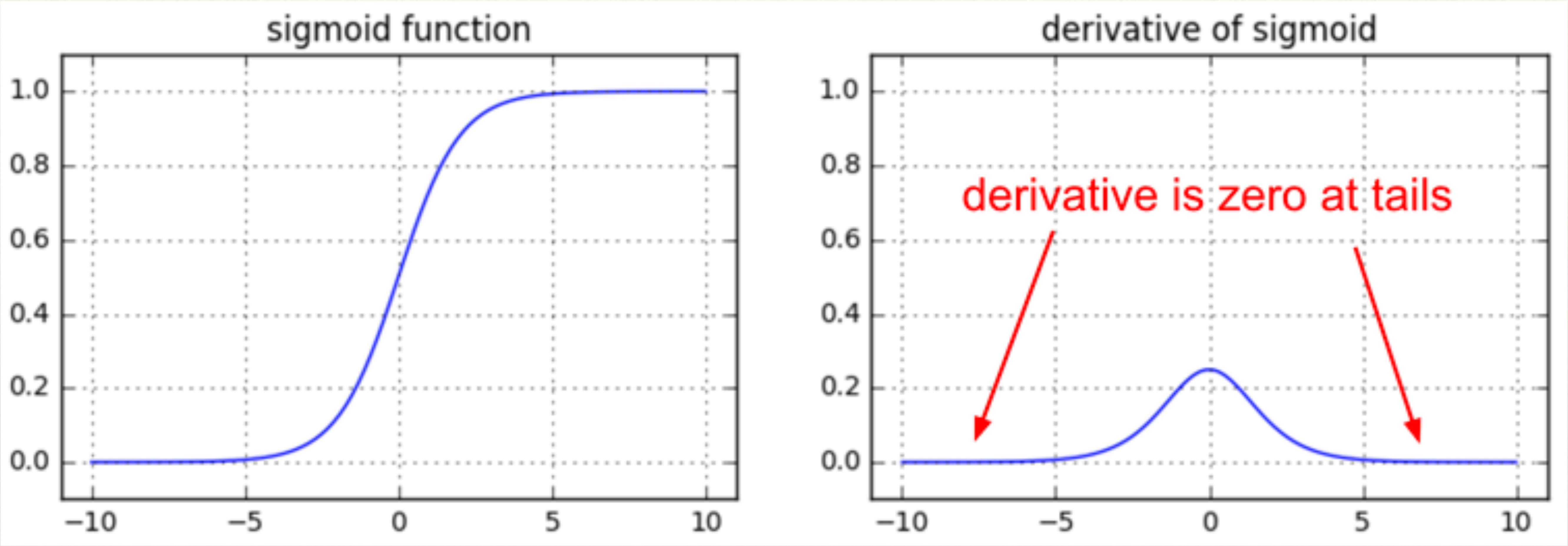
Back Propagation Through Time



Long-term Dependency Issues

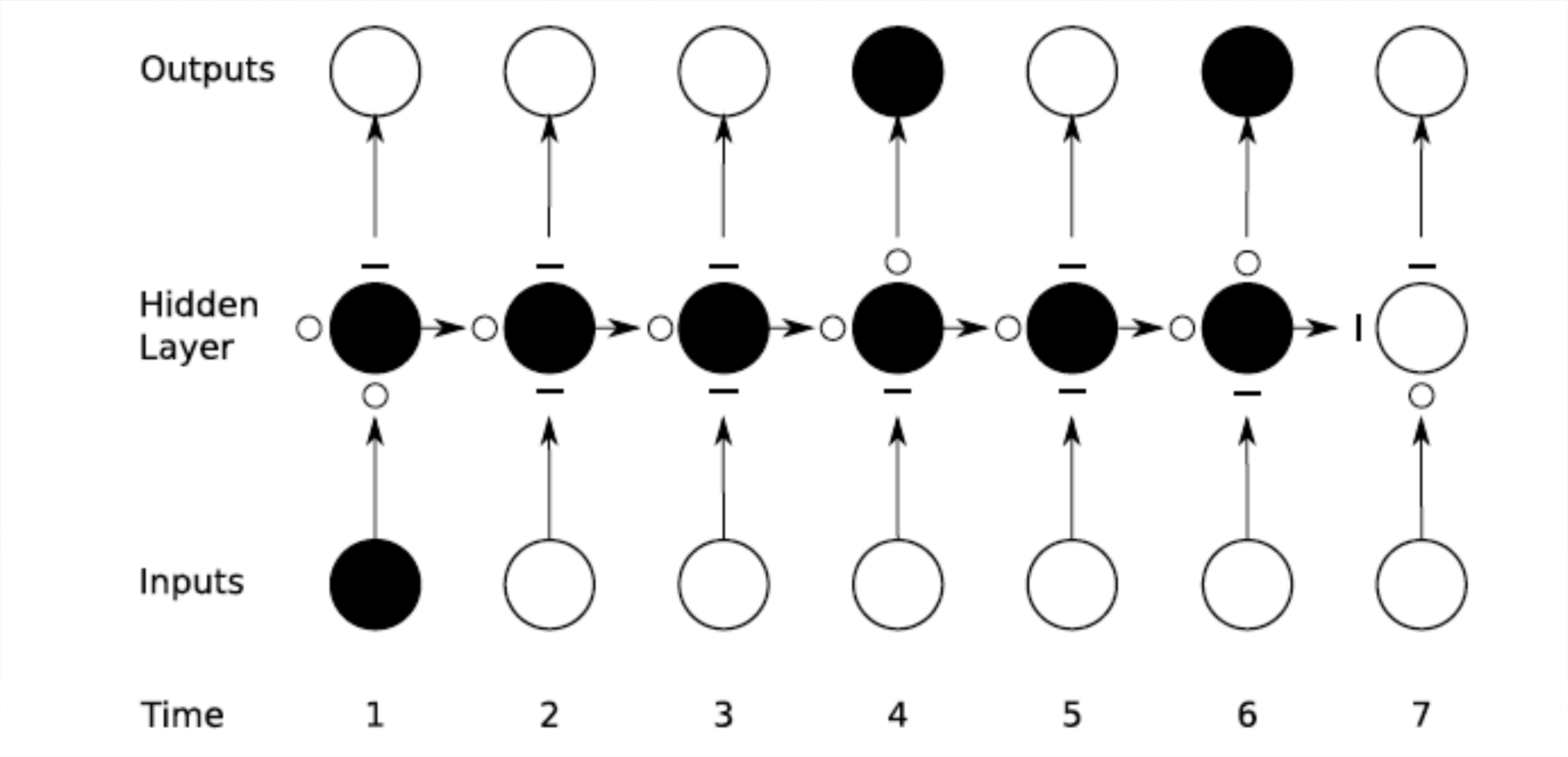


Vanishing/Exploding Gradients

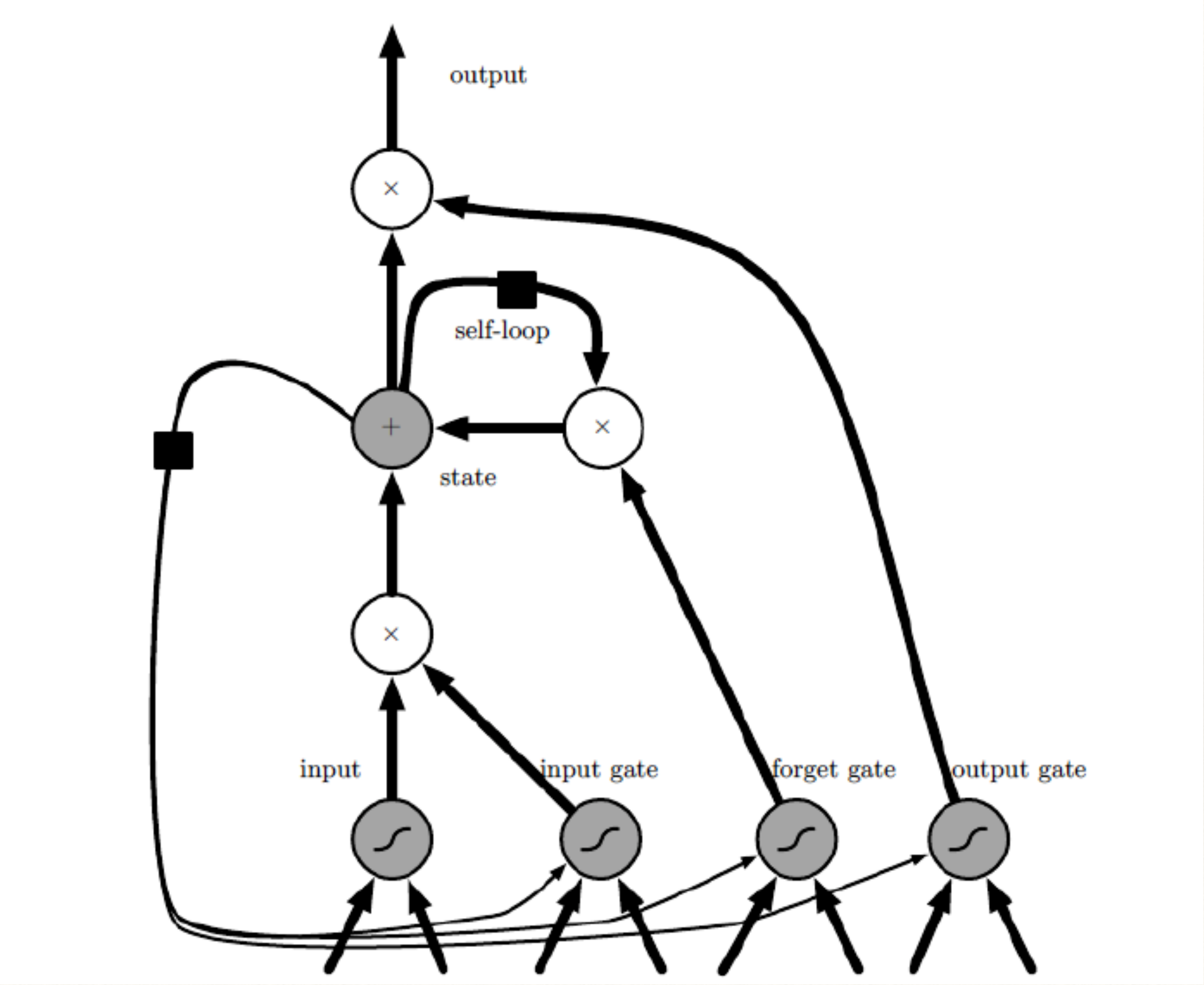


- Gradients either vanish or explode
    - Initial frames may not contribute to gradient computations or may contribute too much.

Long-Short Term Memory



Long Short Term Memory Networks



LSTM Cell

f - sigmoid function
g, h - tanh function

Input Gate

$$a_\iota^t = \sum_{i=1}^{I} w_{i\iota} x_i^t + \sum_{h=1}^{H} w_{h\iota} b_h^{t-1} + \sum_{c=1}^{C} w_{c\iota} s_c^{t-1}$$

$$b_\iota^t = f(a_\iota^t)$$

Forget Gate

$$a_\phi^t = \sum_{i=1}^{I} w_{i\phi} x_i^t + \sum_{h=1}^{H} w_{h\phi} b_h^{t-1} + \sum_{c=1}^{C} w_{c\phi} s_c^{t-1}$$

$$b_\phi^t = f(a_\phi^t)$$

Cell

$$a_c^t = \sum_{i=1}^{I} w_{ic} x_i^t + \sum_{h=1}^{H} w_{hc} b_h^{t-1}$$

$$s_c^t = b_\phi^t s_c^{t-1} + b_\iota^t g(a_c^t)$$

Output Gate

$$a_\omega^t = \sum_{i=1}^{I} w_{i\omega} x_i^t + \sum_{h=1}^{H} w_{h\omega} b_h^{t-1} + \sum_{c=1}^{C} w_{c\omega} s_c^t$$

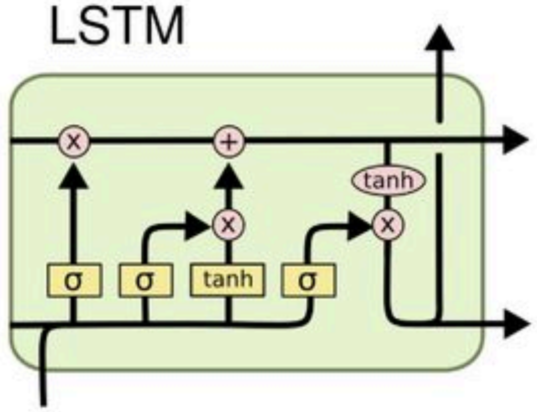
$$b_\omega^t = f(a_\omega^t)$$

LSTM output

$$b_c^t = b_\omega^t h(s_c^t)$$

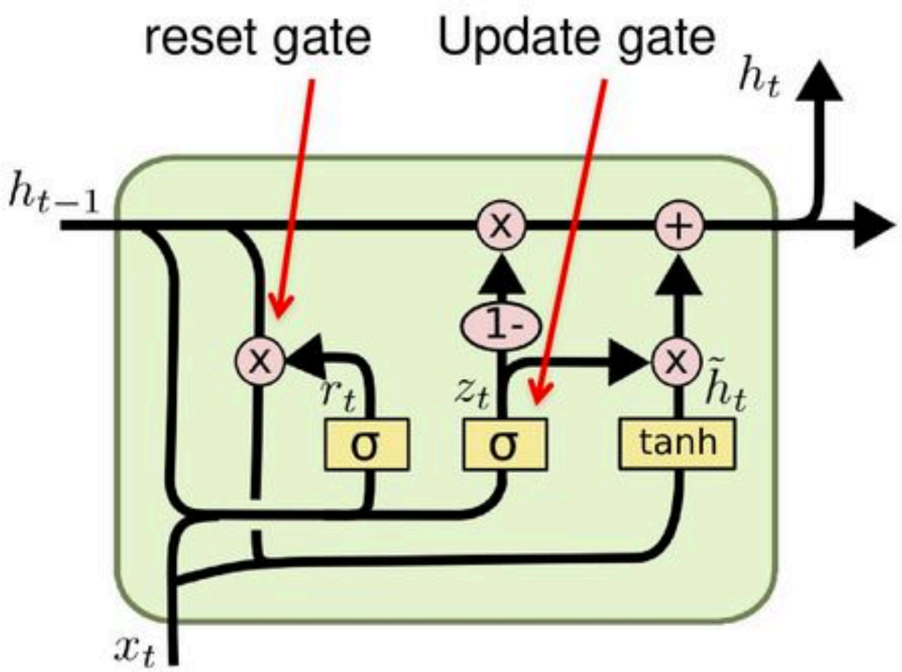
Gated Recurrent Units (GRU)

LSTM



GRU – gated recurrent unit

(more compression)



$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$

$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$

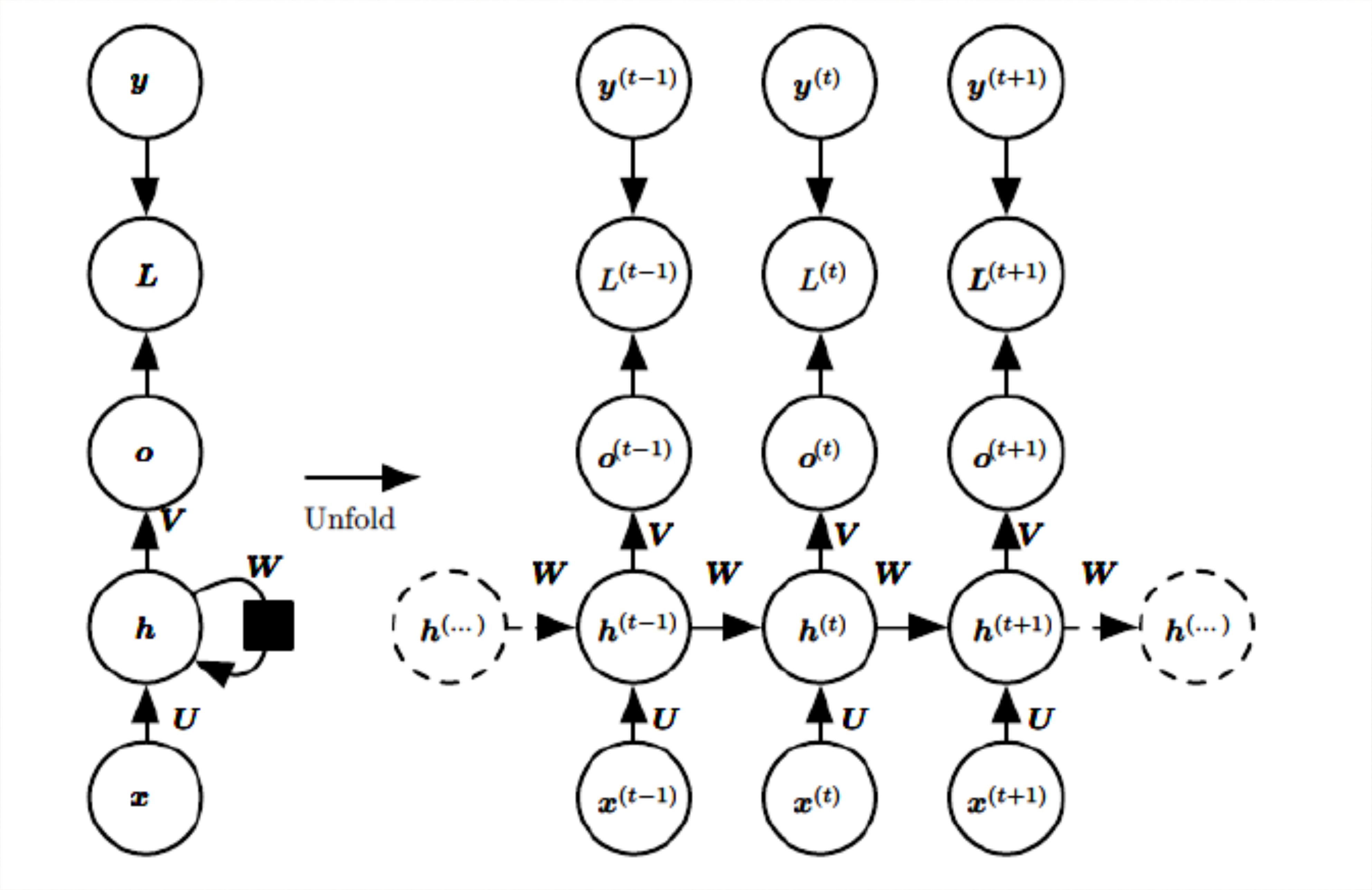
$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

It combines the forget and input into a single update gate.
It also merges the cell state and hidden state. This is simpler than LSTM. There are many other variants too.

X,*: element-wise multiply

Standard Recurrent Networks



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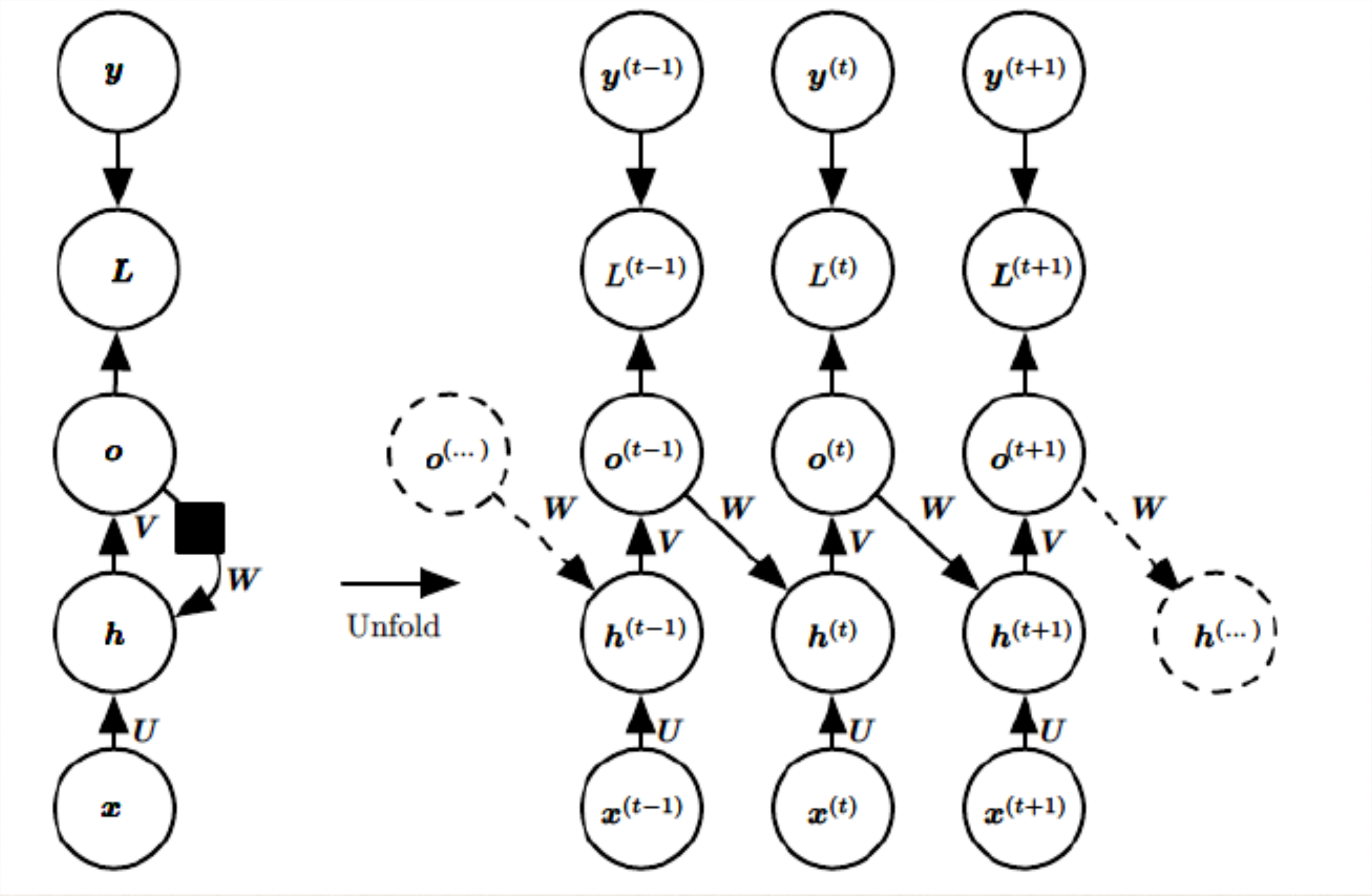
Other Recurrent Networks



Teacher Forcing Networks

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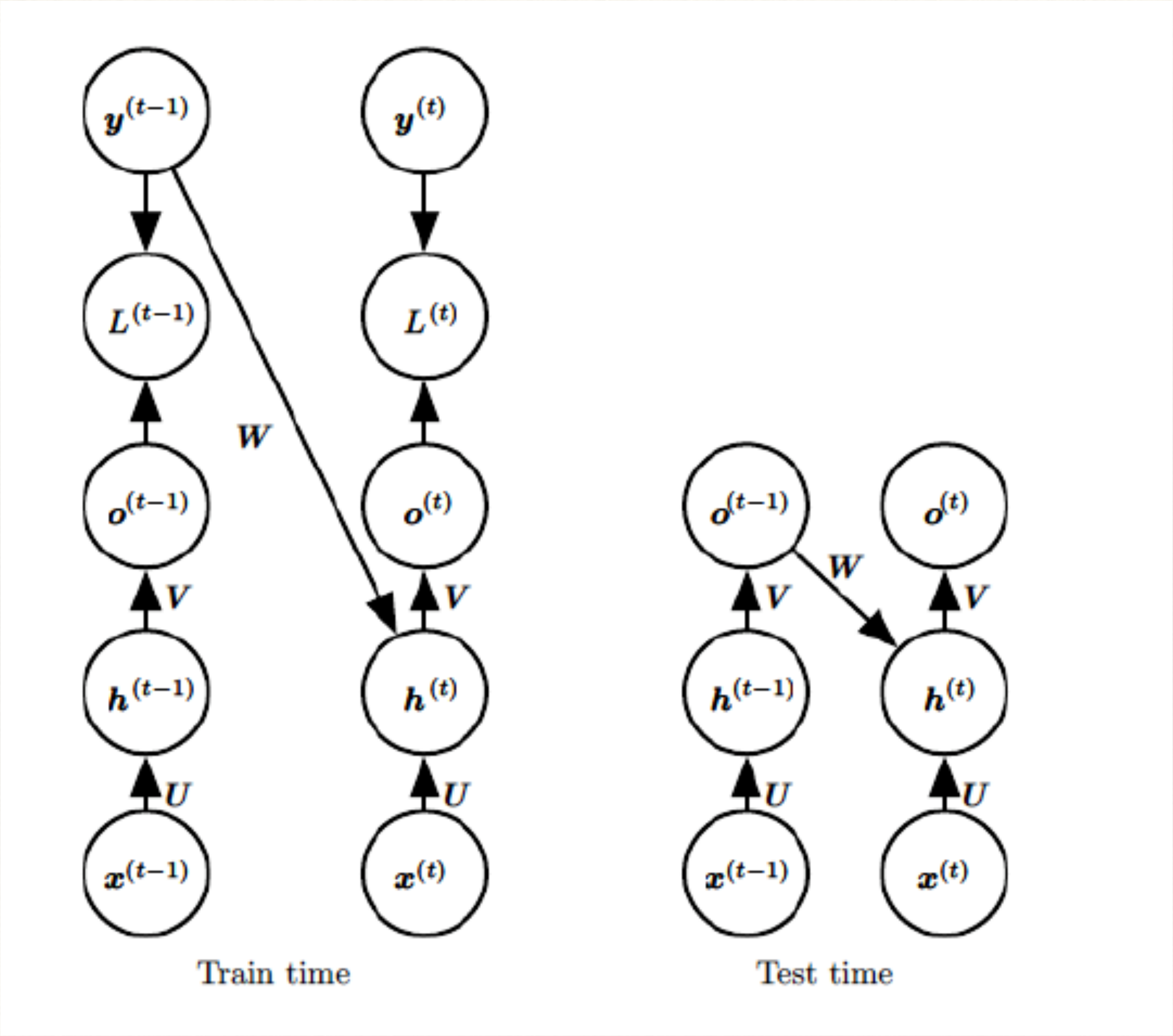
Recurrent Networks



Teacher Forcing Networks

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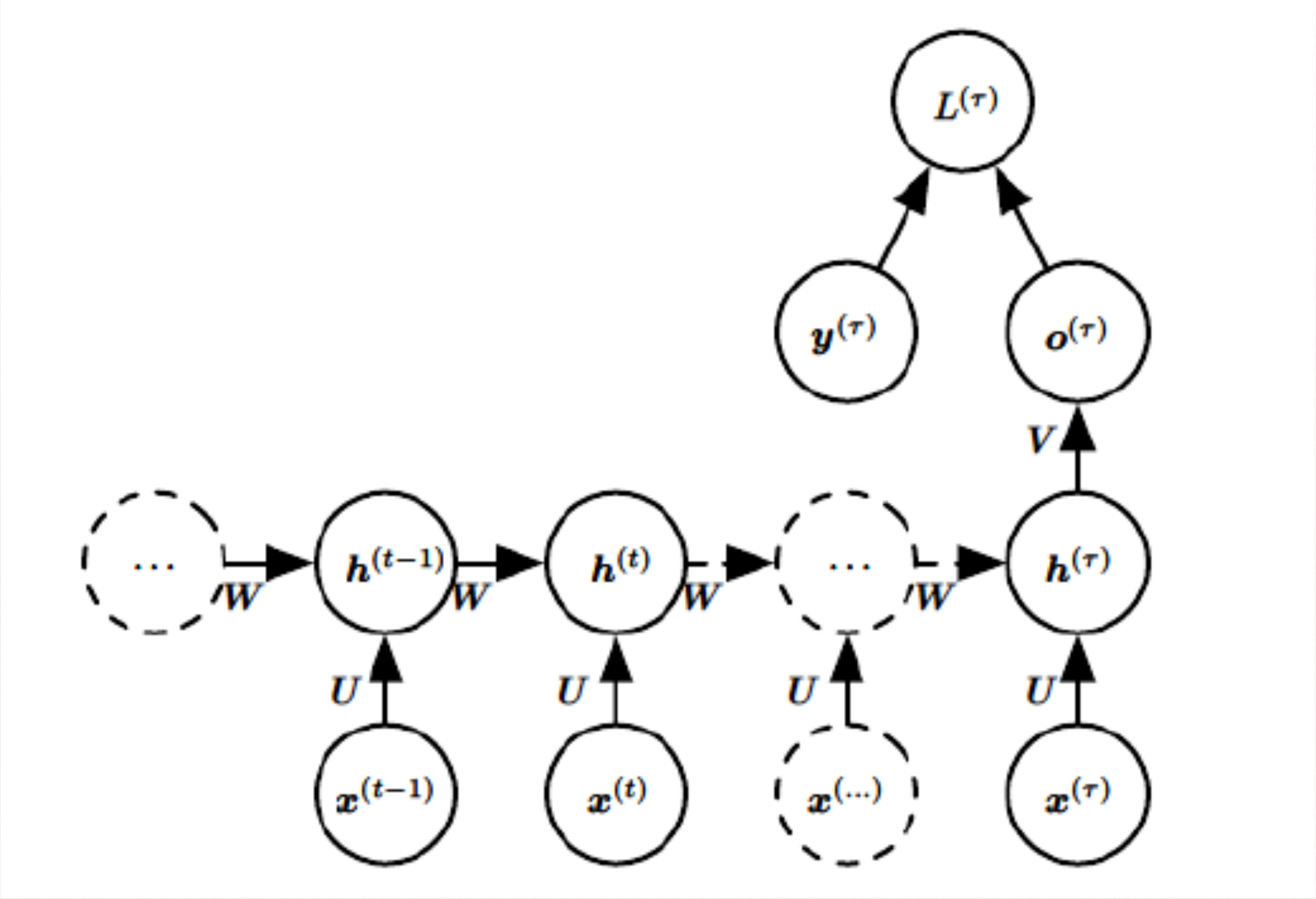
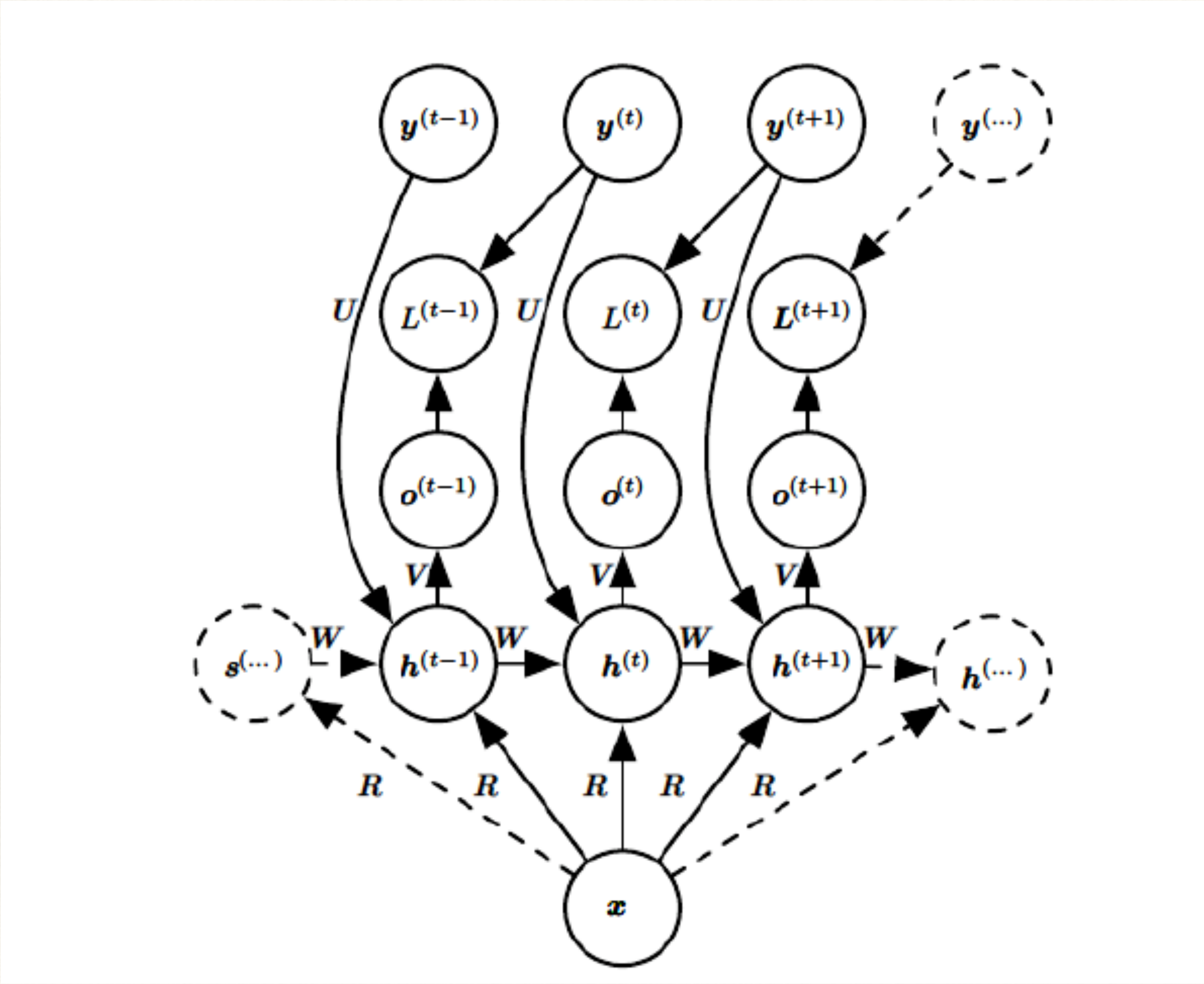
Recurrent Networks



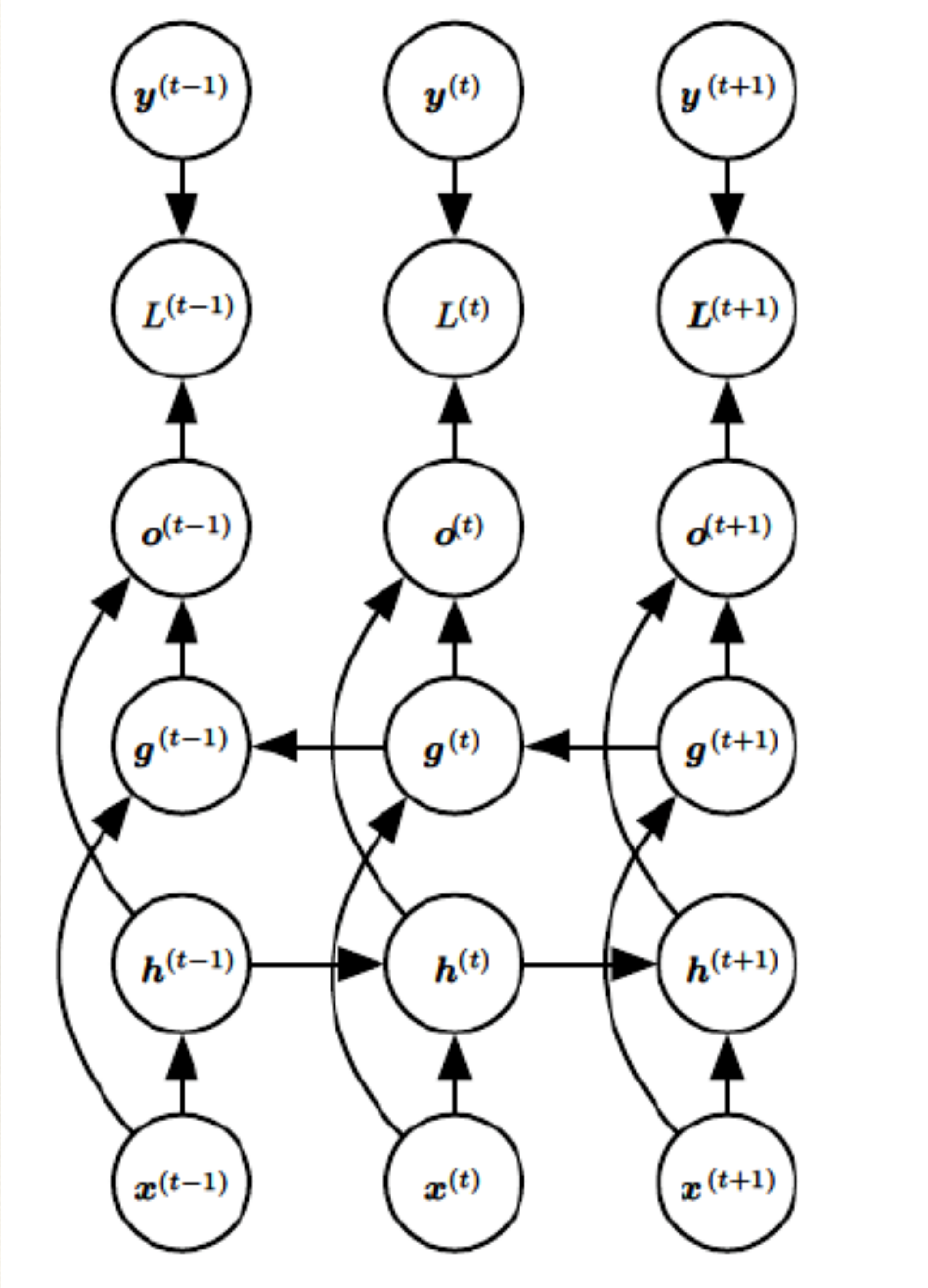
Multiple Input Single Output

Recurrent Networks



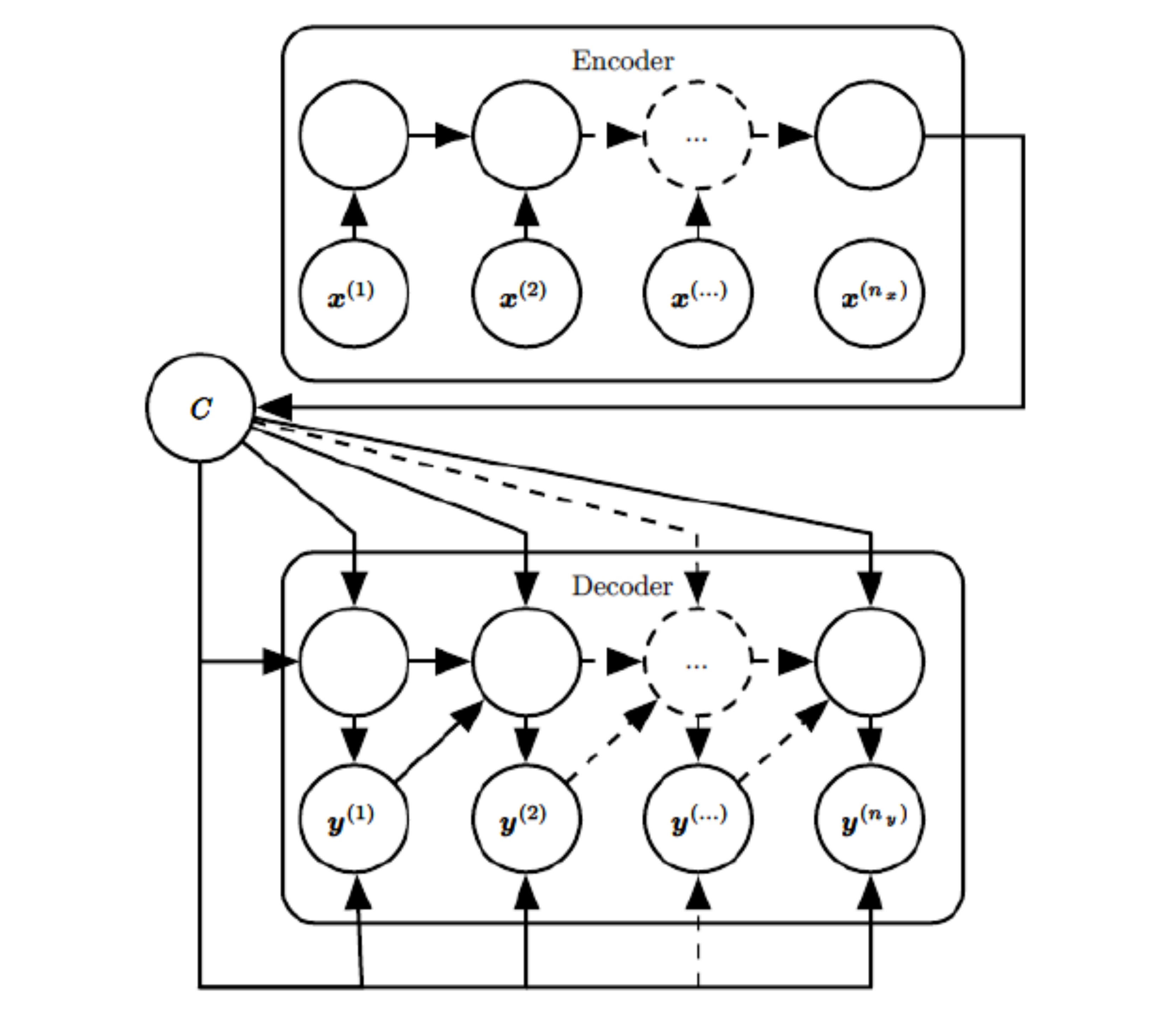
Single Input Multiple Output

Recurrent Networks



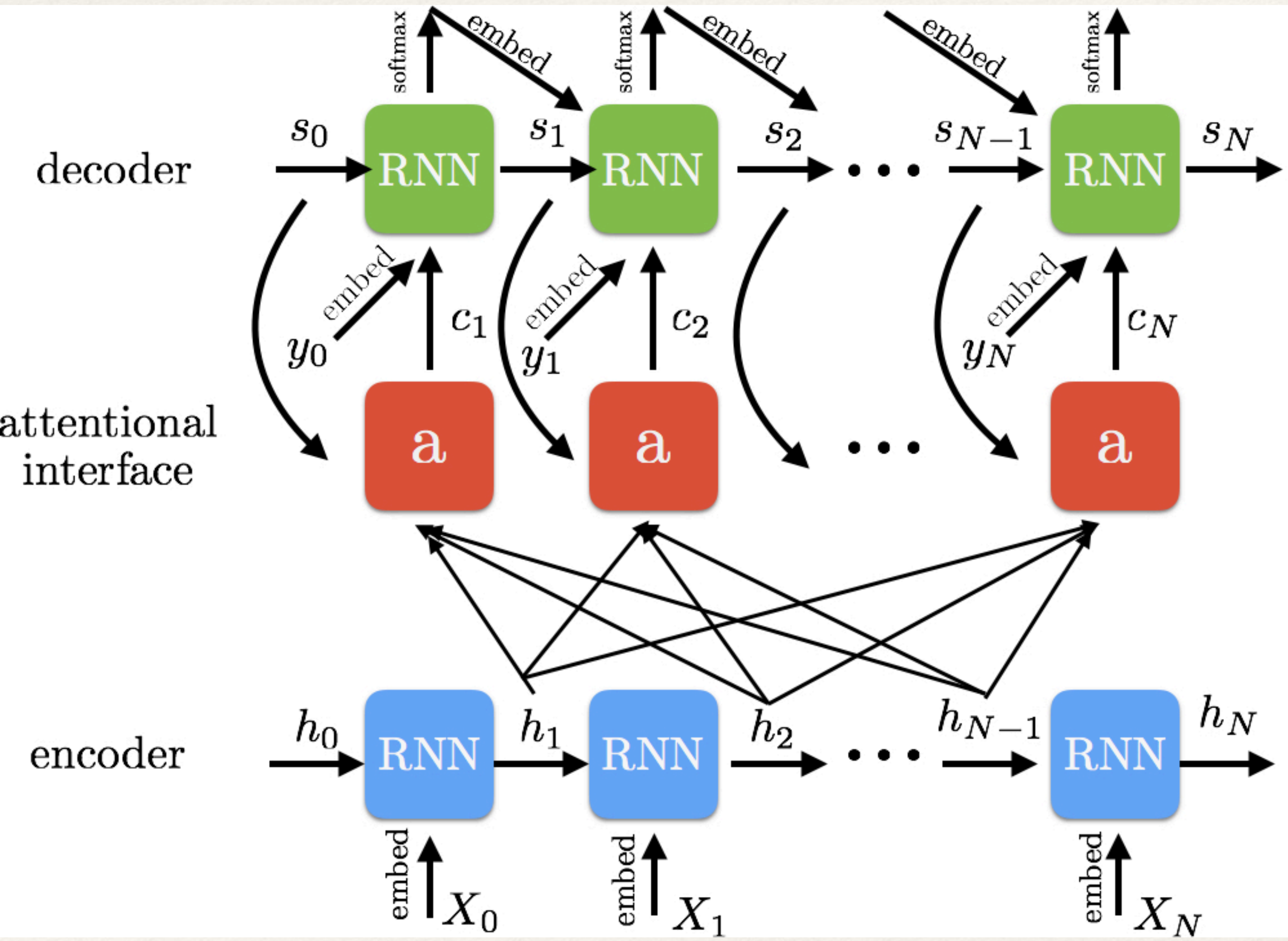
Bi-directional Networks

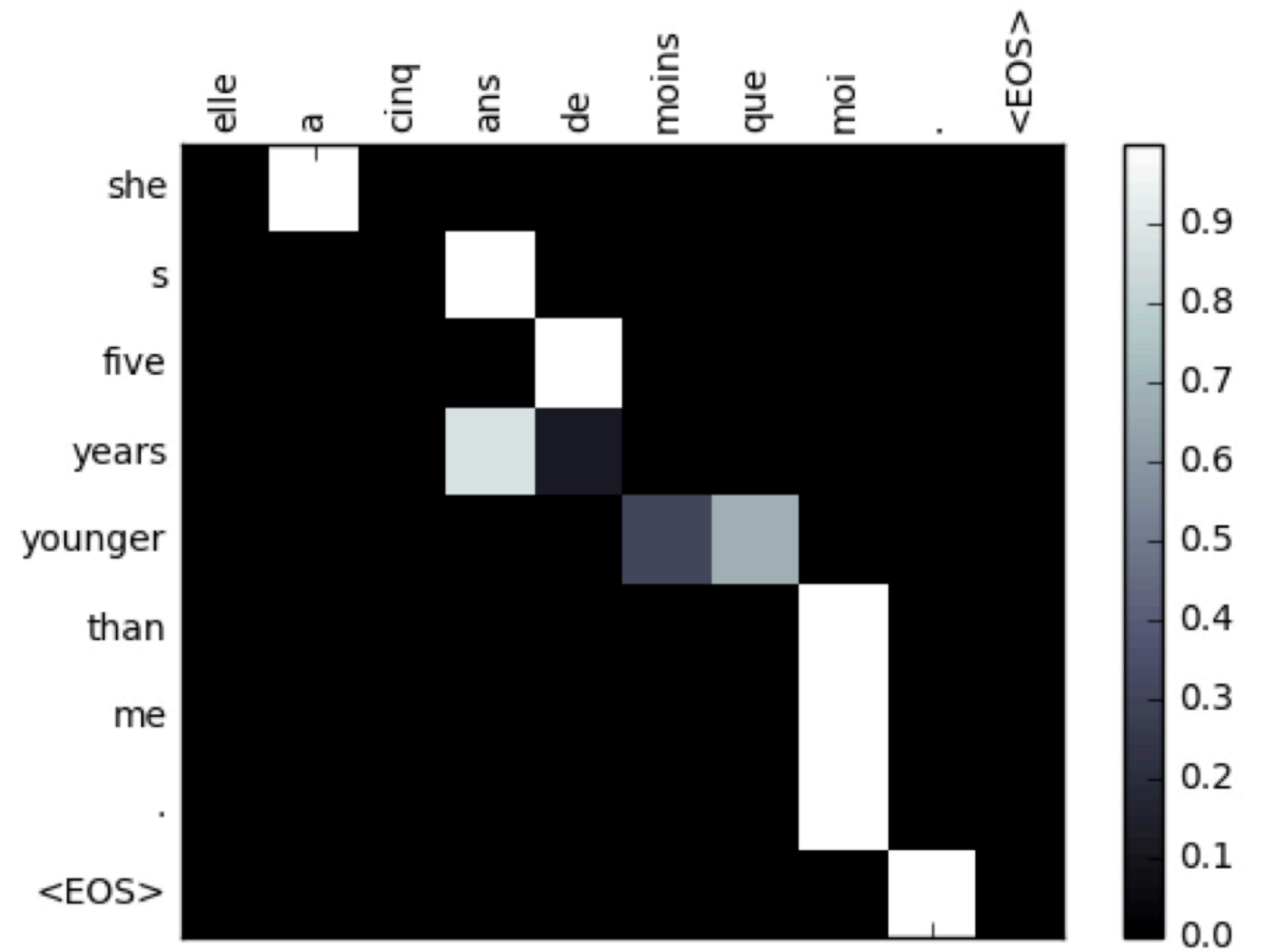
Recurrent Networks



Sequence to Sequence Mapping Networks

Attention Models





Encoder - Decoder Networks with Attention